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The Nambu–Jona–Lasinio model of light nuclei

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Abstract. The Nambu–Jona–Lasinio model of the deuteron suggested by Nambu and Jona–Lasinio (Phys. Rev. 124 (1961) 246) is formulated from the first principles of QCD. The deuteron appears as a neutron–proton collective excitation, i.e. a Cooper np–pair, induced by a phenomenological local four–nucleon interaction in the nuclear phase of QCD. The model describes the deuteron coupled to itself, nucleons and other particles through one–nucleon loop exchanges providing a minimal transfer of nucleon flavours from initial to final nuclear states and accounting for contributions of nucleon–loop anomalies which are completely determined by one–nucleon loop diagrams. The dominance of contributions of nucleon–loop anomalies to effective Lagrangians of low–energy nuclear interactions is justified in the large N_C expansion, where N_C is the number of quark colours.

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1 Introduction

In the beginning of sixties Nambu and Jona–Lasinio suggested a dynamical theory of elementary particles [1,2], the Nambu–Jona–Lasinio (NJL) model, in which nucleons and mesons are derived in a unified way from a fundamental spinor field on the basis of the relativistic extension of the BCS (Bardeen–Cooper–Schrieffer) theory of superconductivity [3]. Nowadays the NJL model has found a great support in the form of the extended Nambu–Jona–Lasinio (ENJL) model with chiral $U(3)\times U(3)$ symmetry as an effective phenomenological approximation of low–energy Quantum Chromodynamics (QCD) [4–6]. Chiral perturbation theory within the ENJL model with a linear and a non–linear realization of chiral $U(3)\times U(3)$ symmetry has been developed in [7,8] and [9], respectively. In the ENJL model mesons are described as $q\bar{q}$ collective excitations, the $q\bar{q}$ Cooper pairs, induced by phenomenological local four–quark interactions. In turn, the low–lying octet and decuplet of baryons can be considered in the NJL approach as three–quark qqq collective excitations produced by phenomenological local six–quark interactions [8]. As has been shown in [4–11] the ENJL model with chiral $U(3)\times U(3)$ symmetry describes at the quark level perfectly well strong low–energy interactions of hadrons in the form of Effective Chiral Lagrangians [12,13].

In parallel to the description of mesons as a collective excitations of a unified spinor field Nambu and Jona–Lasinio suggested to treat the deuteron as a neutron–proton collective excitation, i.e. some kind of a Cooper np–pair [2]. A phenomenological local four–nucleon interaction has been written in the form [2]:

$$\mathcal{L}_{\rm int}(x)=\frac{1}{4}\,g_0\,[\bar{\psi}(x)\gamma_\mu\psi^c(x)\bar{\psi^c}(x)\gamma^\mu\psi(x)+\bar{\psi}(x)\sigma_{\mu\nu}\psi^c(x)\bar{\psi^c}(x)\sigma^{\mu\nu}\psi(x)+\bar{\psi}(x)\gamma_\mu\gamma^5\boldsymbol{\tau}\psi^c(x)\cdot\bar{\psi^c}(x)\gamma^\mu\gamma^5\boldsymbol{\tau}\psi(x)],\quad(1.1)$$

where $\psi(x)$ is a doublet of the nucleon field, then $\psi^c(x)=C\,\bar{\psi}^T(x)$ and $\bar{\psi^c}(x)=\psi^T(x)\,C$, C is a charge conjugation matrix and T is a transposition; $\boldsymbol{\tau}=(\tau^1,\tau^2,\tau^3)$ are the isotopical Pauli matrices, and $\sigma_{\mu\nu}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2$.

According to the Nambu–Jona–Lasinio prescription [2] the phenomenological interactions (1.1) should lead to two bound states: a pseudovector, isoscalar ($J=1^+, I=0$) or the deuteron, where J and I are the total spin and isospin, respectively, coming from the first two interaction terms, and a scalar, isovector ($J=0^+, I=1$) coming from the last term. Unfortunately, Nambu and Jona–Lasinio did not consider the evaluation of the binding energy, the magnetic dipole and electric quadrupole moments of the deuteron in their approach to the deuteron as the Cop-

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per np–pair. Such attempts have been undertaken within the model which has been called the Relativistic field theory model of the deuteron (RFMD) suggested in [14,15]. The RFMD realizes the consideration of the deuteron in the analogous way to the Nambu–Jona–Lasinio approach [2]. For the practical evaluation of the low–energy parameters characterizing the deuteron there has been suggested in the RFMD that the deuteron couples to itself, nucleons and other particles through one–nucleon loop exchanges providing a minimal transfer of nucleon flavours from initial to final nuclear states and accounting for contributions of nucleon–loop anomalies which are completely determined by one–nucleon loop diagrams. Then, there has been argued the dominant role of one–nucleon loop anomalies for the one–nucleon loop exchanges describing strong low–energy nuclear forces. The main problem of the attempts expounded in [14,15] as well as the original idea of the Nambu and Jona–Lasinio to describing the deuteron as Cooper np–pair [2] is in a poor relation to QCD.

In this paper we show that the consideration of the deuteron as a Cooper np–pair induced by a phenomenological local four–nucleon interaction and the description of low–energy couplings of the deuteron to itself, nucleons and other particles through one–nucleon loop exchanges, where nucleon–loop anomalies play the dominant role, is fully motivated by low–energy QCD. The deuteron appears as a Cooper np–pair in the nuclear phase of QCD and couples to itself and other particles through the one–nucleon loop exchanges. The dominance of nucleon–loop anomalies occurs naturally as a consequence of the large N_C expansion, where N_C is the number of quark colour degrees of freedom [16,17]. Nowadays the large N_C expansion suggested by 't Hooft [16] is accepted as a non–perturbative approach of low–energy QCD to the analysis of strong couplings of hadrons and nuclei at low energies [17].

Below we would call the Nambu–Jona–Lasinio approach to the deuteron based on phenomenological local four–nucleon interactions like those given by (1.1) as the Nambu–Jona–Lasinio model of light nuclei or shortly the nuclear Nambu–Jona–Lasinio model with the abbreviation the NNJL model.

The paper is organized as follows. In Sect. 2 we discuss the non–perturbative phases of QCD and formulate the NNJL model from the first principles of QCD. In Sect. 3 we derive the effective Lagrangian for the free deuteron field induced in the nuclear phase of QCD as the neutron–proton collective excitation (the Cooper np–pair) by a phenomenological local four–nucleon interaction. We demonstrate dominant role of one–nucleon loop anomalies for the formation of the effective Lagrangian of the free deuteron field. In Sect. 4 we investigate the electromagnetic properties of the deuteron and derive the effective Lagrangian of the deuteron field coupled to an external electromagnetic field through the magnetic dipole and electric quadrupole moments. We show that the effective Lagrangian of the electromagnetic interactions of the deuteron calculated in the one–nucleon loop approximation at leading order in the large N_C expansion are defined by the anomalies of one–nucleon loop diagrams and have the form of well–known phenomenological electromagnetic interactions introduced by Corben and Schwinger [18] and Aronson [19] for the description of charged vector boson fields coupled to an external electromagnetic field. In the Conclusion we discuss the obtained results.

2 Non–perturbative phases of QCD

The derivation of the NNJL model from the first principles of QCD goes through three non–perturbative phases of the quark–gluon system. We call them as: 1) the low–energy quark–gluon phase (low–energy QCD), 2) the hadronic phase and 3) the nuclear phase.

The low–energy quark–gluon phase of QCD can be obtained by integrating over fluctuations of quark and gluon fields at energies above the scale of spontaneous breaking of chiral symmetry (SBχS) $\Lambda_\chi \simeq 1\,\mathrm{GeV}$. This results in an effective field theory, low–energy QCD, describing strong low–energy interactions of quarks and gluons. The low–energy quark–gluon phase of QCD characterizes itself by the appearance of low–energy gluon–field configurations leading to electric colour fluxes responsible for formation of a linearly rising interquark potential. The former realizes quark confinement.

For the transition to *the hadronic phase of QCD* one should, first, integrate out low–energy gluon degrees of freedom. Integrating over gluon degrees of freedom fluctuating around low–energy gluon–field configurations responsible for formation of a linearly rising interquark potential one arrives at an effective field theory containing only quark (q) and anti–quark ($\bar{q}$) degrees of freedom. This effective field theory describes strong interactions at energies below the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$. The resultant quark system possesses both a chirally invariant and chirally broken phase. In the chirally invariant phase the effective Lagrangian of the quark system is invariant under chiral $U(3) \times U(3)$ group. The chirally invariant phase of the quark system is unstable and the transition to the chirally broken phase is advantageous. The chirally broken phase characterizes itself by three non–perturbative phenomena: SBχS, hadronization (creation of bound quark states with quantum numbers of mesons $q\bar{q}$, $qq\bar{q}\bar{q}$, baryons qqq and so on) and confinement. The transition to the chirally broken phase caused by SBχS accompanies itself with hadronization. Due to quark confinement all observed bound quark states should be colourless. As gluon degrees of freedom are integrated out, in such an effective field theory the entire variety of strong low–energy interactions of hadrons at energies below the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$ is described by quark–loop exchanges.

Since nowadays in continuum space–time formulation of QCD the integration over low–energy gluon–field configurations can be hardly performed explicitly, phenomenological approximations of this integration represented by different effective quark models with chiral $U(3) \times U(3)$ symmetry motivated by QCD are welcomed.

The most interesting effective quark model allowing to describe analytically both SBχS and bosonization (creation of bound $q\bar{q}$ states with quantum numbers of observed low–lying mesons) is the extended Nambu–Jona–Lasinio (ENJL) model [4–11] with linear [7,8,10] and non–linear [9,11] realization of chiral $U(3) \times U(3)$ symmetry. As has been shown in [6] the ENJL model is fully motivated by low–energy multi–colour QCD with a linearly rising interquark potential and N_C quark colour degrees of freedom at $N_C \to \infty$. In the ENJL model mesons are described as $q\bar{q}$ collective excitations (the Cooper $q\bar{q}$–pairs) induced by phenomenological local four–quark interactions. Through one–constituent quark–loop exchanges the Copper $q\bar{q}$–pairs acquire the properties of the observed low–lying mesons such as $\pi(140)$, $K(498)$, $\eta(550)$, $\rho(770)$, $\omega(780)$, $K^*(890)$ and so on. For the description of low–lying octet and decuplet of baryons the ENJL model has been extended by the inclusion of local six–quark interactions responsible for creation of baryons as qqq collective excitations [8].

Integrating then out low–energy quark–field fluctuations, that can be performed in terms of constituent quark–loop exchanges, one arrives at *the hadronic phase of QCD* containing only local meson and baryon fields. The couplings of low–lying mesons and baryons are described by Effective Chiral Lagrangians with chiral $U(3) \times U(3)$ symmetry [4–13].

The nuclear phase of QCD characterizes itself by the appearance of bound nucleon states – nuclei. In order to arrive at *the nuclear phase of QCD* we suggest to start with the hadronic phase of QCD and integrate out heavy hadron degrees of freedom, i.e. all heavy baryon degrees of freedom with masses heavier than masses of low–lying octet and decuplet of baryons and heavy meson degrees of freedom with masses heavier than the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$. At low energies the result of the integration over these heavy hadron degrees of freedom can be represented in the form of phenomenological local many–nucleon interactions. Following the scenario of the hadronic phase of QCD, where hadrons are produced by phenomenological local many–quark interactions as many–quark collective excitations, one can assume that some of these many–nucleon interactions are responsible for creation of many–nucleon collective excitations. These excitations acquire the properties of observed bound nucleon states – nuclei through nucleon–loop and low–lying meson exchanges. This results in an effective field theory describing nuclei and their low–energy interactions in analogy with Effective Chiral Lagrangian approaches [12,13]. Chiral perturbation theory can be naturally incorporated into this effective field theory of low–energy interactions of nuclei.

We would like to emphasize that in this scenario of the quantum field theoretic formation of nuclei and their low–energy interactions nuclei are considered as elementary particles described by local interpolating fields. In parallel to the Nambu–Jona–Lasinio approach to light nuclei [2] the representation of nuclei as elementary particles has been suggested by Sakita and Goebal [20] and Kim and Primakoff [21] for the description of electromagnetic and weak nuclear processes. We develop the quantum field theoretic approach to the interpretation of nuclei as elementary particles represented by local interpolating fields by starting with QCD.

In this scenario the deuteron, being the lightest bound nucleon state, appears in the nuclear phase of QCD as the neutron–proton collective excitation (the Cooper np–pair) induced by a phenomenological local four–nucleon interaction caused by the contributions of heavy hadron exchanges at low energies. Through one–nucleon loop exchanges the Cooper np–pair with quantum numbers of the physical deuteron acquires the properties of the physical deuteron (i) the binding energy $\varepsilon_{\mathrm{D}} = 2.225\,\mathrm{MeV}$, (ii) the magnetic dipole moment $\mu_{\mathrm{D}} = 0.857\,\mu_{\mathrm{N}}$, where μ_{N} is a nuclear magneton, (iii) the electric quadrupole moment $Q_{\mathrm{D}} = 0.286\,\mathrm{fm}^2$ [22] and so on.

We would like to emphasize that Sakita and Goebal [20] by treating the deuteron as an elementary particle described by a local interpolating field $D_\mu(x)$ have calculated the cross section for the photo–disintegration of the deuteron γ + D $\to$ n + p within the dispersion relation approach. The more recent analysis of the same process by using the dispersion relations has been carried out by Anisovich and Sadovnikova [23] based on the dispersion relation technique developed by Anisovich *et al.* [24]. The dispersion relation approach as well as the NNJL model is a relativistically covariant one. Within the dispersion relation approach one deals with directly the amplitudes of the process of the deuteron coupled to other particles keeping under the control intermediate states in the form of the pole and branching point singularities. The residues at pole singularities are defined by the effective coupling constants which as usual in the dispersion relation technique are taken from experimental data. Unlike the dispersion relation approach in the NNJL model developing the Lagrange approach to nuclear forces we focus on the evaluation of the effective coupling constants via the derivation of the effective Lagrangians of the deuteron coupled to nucleons and other particles at low energies. These effective coupling constants are defined in the NNJL model by one–nucleon loop anomalies related to high–energy $N\bar{N}$ fluctuations of virtual nucleon (N) and anti–nucleon ($\bar{N}$) fields. Thus, the dispersion relation approach to the description of low–energy interactions of the deuteron and the NNJL model do not contradict but should complement each other.

In the form of the path integral formulation of QCD the non–perturbative phases of QCD can be represented by the following sequence of transformations. Let us start with the path integral over the quark q, anti–quark $\bar{q}$ and gluon A fields related to a generating functional of quark and gluon Green functions and defined by

$$\mathcal{Z} = \int Dq D\bar{q} DA\, e^{i\int d^4x\, \mathcal{L}^{\mathrm{QCD}}[\bar{q}, q, A]}. \qquad (2.1)$$

Integrating over high–energy quark–gluon fluctuations restricted from below by the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$ we arrive at the generating functional

$$\mathcal{Z} = \int DqD\bar{q}Da\, e^{i\int d^4x\, \mathcal{L}^{\rm QCD}_{\rm eff}[\bar{q}, q, \tilde{A} + a]} \qquad (2.2)$$

describing strong low–energy interactions of quarks and gluons in *the low–energy quark–gluon phase of QCD*, where $\tilde{A}$ and a are non–perturbative gluon–field configurations responsible for the formation of a linearly rising interquark potential providing quark confinement and the gluon–field fluctuations around these gluon–field configurations.

Integrating then over the gluon–field fluctuations a we obtain the generating functional

$$\mathcal{Z} = \int DqD\bar{q}\, e^{i\int d^4x\, \mathcal{L}_{\rm eff}[\bar{q}, q, local\ multi-q\ couplings]}. \qquad (2.3)$$

This generating functional describes the effective theory of quarks coupled to each other at energies of order $\Lambda_\chi \simeq 1\,GeV$ and less. At the phenomenological level the result of the integration over gluon–field configurations can be represented in the form of phenomenological local multi–quark interactions responsible for the creation of multi–quark collective excitations. This quark system is unstable under SBχS and hadronization. By converting quark degrees of freedom into the hadronic ones or differently hadronizing the quark system we arrive at the generating functional given by the path integral over hadronic degrees of freedom only

$$\mathcal{Z} = \int DM_\ell DB_\ell DM_h DB_h\, e^{i\int d^4x\, \mathcal{L}_{\rm eff}[M_\ell, B_\ell, M_h, B_h]}, \qquad (2.4)$$

where M_ℓ, B_ℓ and M_h, B_h are local interpolating fields of mesons and baryons. The indices ℓ and h correspond to light hadrons with masses of order of 1 GeV and less and heavy hadrons with masses much greater than 1 GeV. For the practical applications to the description of low–energy couplings of light and heavy hadrons the effective Lagrangian $\mathcal{L}_{\rm eff}[M_\ell, B_\ell, M_h, B_h]$ can be approximated by Effective Chiral Lagrangians for light hadrons [12,13] and heavy hadrons [25] as well. The generating functional (2.4) describes low–energy interactions of hadrons in *the hadronic phase of QCD*.

Integrating over heavy baryon degrees of freedom given by the fields M_h and B_h we get the generating functional in the form of the path integral over the light hadron degrees of freedom

$$\mathcal{Z} = \int DM_\ell \times DB_\ell\, e^{i\int d^4x \mathcal{L}_{\rm eff}[M_\ell, B_\ell, local\ multi-B_\ell\ couplings]}. \qquad (2.5)$$

At low energies the result of the integration over heavy hadron degrees of freedom can be represented phenomenologically by local multi–baryon couplings some of which should be responsible for the creation of multi–baryon excitations with quantum numbers of nuclei. In term of the local interpolating fields of nuclei the generating functional (2.5) acquires the form

$$\mathcal{Z} = \int DM_\ell\, DB_\ell\, D{\rm D}\, D\,{}^3{\rm H}\, D\,{}^3{\rm He}\, D\,{}^4{\rm He}\ldots \times e^{i\int d^4x\, \mathcal{L}_{\rm eff}[M_\ell, B_\ell, {\rm D}, {}^3{\rm H}, {}^3{\rm He}, {}^4{\rm He}, \ldots]}, \qquad (2.6)$$

where D, ^{3}H, ^{3}He and ^{4}He are the local interpolating fields of the deuteron, the triton, the helium–3 and the helium–4, respectively. The ellipses stand for a possible contribution of other nuclei. The generating functional (2.6) describes *the nuclear phase of QCD*, when nuclei couple to each other and light hadrons at low energies. Chiral perturbation theory [7,8] is naturally incorporated into this theory.

3 The deuteron as a Cooper np–pair

In order to describe the deuteron as a Cooper np–pair we introduce a phenomenological local four–nucleon interaction caused by the integration over heavy hadron degrees of freedom. First, let us consider the simplest form of this local four–nucleon interaction

$$\mathcal{L}_{\rm int}(x) = -\frac{g^2_{\rm V}}{4M^2_{\rm N}}\, j^\dagger_\mu(x) j^\mu(x), \qquad (3.1)$$

where $g_{\rm V}$ is the phenomenological coupling constant of the NNJL model [14,15], $M_{\rm N} = 940\,{\rm MeV}$ is the nucleon mass. We neglect here the electromagnetic mass difference for the neutron and the proton. As has been found in [14,15] the coupling constant $g_{\rm V}$ is related to the electric quadrupole moment of the deuteron $Q_{\rm D}$: $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$ [15].

The nucleon current $j^\mu(x)$ with the quantum numbers of the deuteron is defined by [14,15]

$$j^\mu(x) = -i\,[\bar{p^c}(x)\gamma^\mu n(x) - \bar{n^c}(x)\gamma^\mu p(x)]. \qquad (3.2)$$

Here $p(x)$ and $n(x)$ are the interpolating fields of the proton and the neutron, $N^c(x) = C\,\bar{N}^T(x)$ and $\bar{N}^c(x) = N^T(x)\,C$, where C is a charge conjugation matrix and T is a transposition. In terms of the electric quadrupole moment of the deuteron the phenomenological local four–nucleon interaction (3.1) reads

$$\mathcal{L}_{\rm int}(x) = -\frac{1}{2}\,\pi^2\, Q_{\rm D}\, j^\dagger_\mu(x) j^\mu(x). \qquad (3.3)$$

Now let us discuss the behaviour of the phenomenological coupling constant $g^2_{\rm V}/4M^2_{\rm N}$ from the point of view of the large N_C expansion in QCD with the $SU(N_C)$ gauge group at $N_C \to \infty$ [16,17]. Suppose, for simplicity, that the phenomenological four–nucleon interaction (3.1) is caused by exchanges of the scalar $f_0(980)$ and $a_0(980)$ mesons being the lightest states among heavy hadrons we have integrated out.

Through a linear realization of chiral $U(3)\times U(3)$ symmetry and the Goldberger–Treiman relation one can find that the coupling constant of σ–mesons $g_{\sigma\mathrm{NN}}$, the $q\bar{q}$–scalar mesons, coupled to the octet of low–lying baryons should be of order $g_{\sigma\mathrm{NN}} \sim O(\sqrt{N_C})$ at $N_C \to \infty$. The scalar mesons $f_0(980)$ and $a_0(980)$ are most likely four–quark states with $qq\bar{q}\bar{q}$ quark structure [26,27]. In the limit $N_C \to \infty$ such $qq\bar{q}\bar{q}$ states are suppressed by a factor $1/N_C$ [17]. Thus, an effective coupling constant of low–energy NN interaction caused by the $qq\bar{q}\bar{q}$ scalar meson exchanges should be of order $O(1/N_C)$ at $N_C \to \infty$. By taking into account that in QCD with $N_C \to \infty$ the nucleon mass M_{N} is proportional to N_C [17], $M_{\mathrm{N}} = N_C M_q$, where $M_q \sim 300\,\mathrm{MeV}$ is the constituent quark mass, we can introduce the nucleon mass M_{N} in the effective coupling constant as a dimensional parameter absorbing the factor N_C^2, i.e. $g^2_{\mathrm{V}}/4M^2_{\mathrm{N}}$. This is also required by the correct dependence of the deuteron mass on N_C. As a result the phenomenological coupling constant g_{V} turns out to be of order $O(\sqrt{N_C})$ at $N_C \to \infty$.

We should emphasize that one does not need to know too much about quark structure of heavy hadron degrees of freedom we have integrated out. Without loss of generality one can argue that among the multitude of contributions caused by the integration over heavy hadron degrees of freedom one can always find the required local four–nucleon interaction the effective coupling constant of which behaves like $O(1/N_C)$ at $N_C \to \infty$. As we show below this behaviour of the coupling constant of the phenomenological four–nucleon interaction leads to the deuteron as bound neutron–proton state, and it is also consistent with the large N_C dependence of low–energy parameters of the physical deuteron [17].

The effective Lagrangian of the np–system unstable under creation of the Cooper np–pair with quantum numbers of the deuteron is then defined by

$$\begin{aligned}\mathcal{L}^{\mathrm{np}}(x) &= \bar{n}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\mathrm{N}})\,n(x)\\ &\quad+\bar{p}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\mathrm{N}})\,p(x)\\ &\quad-\frac{g^2_{\mathrm{V}}}{4M^2_{\mathrm{N}}}\,j^{\dagger}_{\mu}(x)j^{\mu}(x) + \ldots,\end{aligned} \tag{3.4}$$

where ellipses stand for low–energy interactions of the neutron and the proton with other fields.

In order to introduce the interpolating local deuteron field we should linearalize the Lagrangian (3.4). Following the procedure described in [4–11] for the inclusion of local interpolating meson fields in the ENJL model we get

$$\begin{aligned}&\mathcal{L}^{\mathrm{np}}(x) \to \bar{n}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\mathrm{N}})\,n(x)\\ &+\bar{p}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\mathrm{N}})\,p(x)\\ &+M^2_0\,D^{\dagger}_{\mu}(x)D^{\mu}(x) + g_{\mathrm{V}}j^{\dagger}_{\mu}(x)D^{\mu}(x)\\ &+g_{\mathrm{V}}j^{\mu}(x)D^{\dagger}_{\mu}(x) + \ldots,\end{aligned} \tag{3.5}$$

where $M_0 = 2\,M_{\mathrm{N}}$ and $D^{\mu}(x)$ is a local interpolating field with quantum numbers of the deuteron.

In order to derive the effective Lagrangian of the physical deuteron field we should integrate over nucleon fields in the one–nucleon loop approximation [2,14,15]. The one–nucleon loop approximation of low–energy nuclear forces allows (i) to transfer nucleon flavours from an initial to a final nuclear state by a minimal way and (ii) to take into account contributions of nucleon–loop anomalies [28–31], which are fully defined by one–nucleon loop diagrams [29–31]. It is well–known that quark–loop anomalies play an important role for the correct description of strong low–energy interactions of low–lying hadrons [4–13]. We argue the dominant role of nucleon–loop anomalies for the correct description of low–energy nuclear forces in nuclear physics. We demonstrate below the dominant role of nucleon–loop anomalies by example of the evaluation of the effective Lagrangian of the free deuteron field.

The effective Lagrangian of the free deuteron field evaluated in the one–nucleon loop approximation is defined by [14,15]:

$$\begin{aligned}\int d^4x\,\mathcal{L}_{\mathrm{eff}}(x) &= \int d^4x\,M^2_0\,D^{\dagger}_{\mu}(x)D^{\mu}(x)\\ &-\int d^4x\int\frac{d^4x_1 d^4k_1}{(2\pi)^4}\,e^{-ik_1\cdot(x - x_1)}\\ &\times D^{\dagger}_{\mu}(x)D_{\nu}(x_1)\,\frac{g^2_{\mathrm{V}}}{4\pi^2}\,\Pi^{\mu\nu}(k_1;Q),\end{aligned} \tag{3.6}$$

where the structure function $\Pi^{\mu\nu}(k_1;Q)$ is given by

$$\Pi^{\mu\nu}(k_1;Q) = \int\frac{d^4k}{\pi^2 i}\mathrm{tr}\Big\{\frac{1}{M_{\mathrm{N}} - \hat{k} - \hat{Q} - \hat{k}_1}\gamma^{\mu}\frac{1}{M_{\mathrm{N}} - \hat{k} - \hat{Q}}\gamma^{\nu}\Big\}. \tag{3.7}$$

The 4–momentum $Q = a\,k_1$ is an arbitrary shift of momenta of virtual nucleon fields with an arbitrary parameter a. According to [29,31] the Q–dependent parts of one–nucleon loop diagrams are related to the anomalies of these diagrams, thereby, the correct evaluation of the Q–dependence of one–nucleon loop diagrams is a great deal of importance in the NNJL model stating a dominant role of nucleon–loop anomalies. For the evaluation of the Q–dependence of the structure function $\Pi^{\mu\nu}(k_1;Q)$ we apply the procedure invented by Gertsein and Jackiw [29] (see also [15]):

$$\begin{aligned}&\Pi^{\mu\nu}(k_1;Q) - \Pi^{\mu\nu}(k_1;0) = \int^1_0 dx\,\frac{d}{dx}\,\Pi^{\mu\nu}(k_1;xQ)\\ &= \int^1_0 dx\int\frac{d^4k}{\pi^2 i}Q^{\lambda}\frac{\partial}{\partial k^{\lambda}}\mathrm{tr}\Big\{\frac{1}{M_{\mathrm{N}} - \hat{k} - x\hat{Q} - \hat{k}_1}\gamma^{\mu}\\ &\qquad\times\frac{1}{M_{\mathrm{N}} - \hat{k} - x\hat{Q}}\gamma^{\nu}\Big\} =\\ &= 2\int^1_0 dx\lim_{k\to\infty}\Big\langle\frac{Q\cdot k}{k^2}\mathrm{tr}\{(M_{\mathrm{N}} + \hat{k} + x\hat{Q} + \hat{k}_1)\gamma^{\mu}\\ &\qquad\times|,(M_{\mathrm{N}} + \hat{k} + x\hat{Q})\gamma^{\nu}\}\Big\rangle\\ &= 2\,(2Q^{\mu}Q^{\nu} - Q^2\,g^{\mu\nu}) + 2(k^{\mu}_1 Q^{\nu} + k^{\nu}_1 Q^{\mu} - k_1\cdot Q\,g^{\mu\nu})\\ &= -2\,a(a+1)\,(k^2_1\,g^{\mu\nu} - 2\,k^{\mu}_1 k^{\nu}_1).\end{aligned} \tag{3.8}$$

Thus, we obtain

$$\Pi^{\mu\nu}(k_1;Q) - \Pi^{\mu\nu}(k_1;0) = -2\,a(a+1) \times (k_1^2\,g^{\mu\nu} - 2\,k_1^{\mu}k_1^{\nu}). \quad (3.9)$$

We would like to emphasize that the r.h.s. of (3.9) is an explicit expression completely defined by high–energy (short–distance) $N\bar{N}$ fluctuations, since the virtual momentum k is taken at the limit $k \to \infty$, and related to the anomaly of the one–nucleon loop diagram with two vector vertices (the VV–diagram) [29,31].

The structure function $\Pi^{\mu\nu}(k_1;0)$ has been evaluated in [14,15] and reads

$$\Pi^{\mu\nu}(k_1;0) = \frac{4}{3}(k_1^2 g^{\mu\nu} - k_1^{\mu}k_1^{\nu})J_2(M_{\rm N}) + 2g^{\mu\nu}[J_1(M_{\rm N}) + M_{\rm N}^2 J_2(M_{\rm N})], \quad (3.10)$$

where $J_1(M_{\rm N})$ and $J_2(M_{\rm N})$ are the quadratically and logarithmically divergent integrals [14,15]:

$$J_1(M_{\rm N}) = \int\frac{d^4k}{\pi^2 i}\frac{1}{M_{\rm N}^2 - k^2} = 4\int_0^{\Lambda_{\rm D}}\frac{d|\boldsymbol{k}|\boldsymbol{k}^2}{(M_{\rm N}^2 + \boldsymbol{k}^2)^{1/2}},$$

$$J_2(M_{\rm N}) = \int\frac{d^4k}{\pi^2 i}\frac{1}{(M_{\rm N}^2 - k^2)^2} = 2\int_0^{\Lambda_{\rm D}}\frac{d|\boldsymbol{k}|\boldsymbol{k}^2}{(M_{\rm N}^2 + \boldsymbol{k}^2)^{3/2}}. \quad (3.11)$$

The cut–off $\Lambda_{\rm D}$ restricts from above 3–momenta of low–energy fluctuations of virtual neutron and proton fields forming the physical deuteron [14,15]. As has been shown in [14,15] the cut–off $\Lambda_{\rm D}$ is much less than the mass of the nucleon, $M_{\rm N} \gg \Lambda_{\rm D}$ [14,15]. This leads to the relation between the divergent integrals:

$$J_1(M_{\rm N}) = 2\,M_{\rm N}^2\,J_2(M_{\rm N}) = \frac{4}{3}\frac{\Lambda_{\rm D}^3}{M_{\rm N}} \sim O(1/N_C) \quad (3.12)$$

which we use below. Note that in (3.10) we have taken into account only the leading terms in the external momentum expansion, i.e. the k_1–expansion [14,15].

The justification of the dominance of the leading order contributions in expansion in powers of external momenta can be provided in the large N_C approach to the description of QCD in the non–perturbative regime. Indeed, in QCD with the $SU(N_C)$ gauge group at $N_C \to \infty$ the baryon mass is proportional to the number of quark colours [17]: $M_{\rm N} \sim N_C$. Since for the derivation of effective Lagrangians describing the deuteron itself and amplitudes of processes of low–energy interactions of the deuteron coupled to other particles all external momenta of interacting particles should be kept off–mass shell, the masses of virtual nucleon fields taken at $N_C \to \infty$ are larger compared with external momenta. By expanding one–nucleon loop diagrams in powers of $1/M_{\rm N}$ we get an expansion in powers of $1/N_C$. Keeping the leading order in the large N_C expansion we are leaving with the leading order contributions in an external momentum expansion. We should emphasize that anomalous contributions of one–nucleon loop diagrams are defined by the least powers of an external momentum expansion. This implies that in the NNJL model effective Lagrangians of low–energy interactions are completely determined by contributions of one–nucleon loop anomalies. The divergent contributions having the same order in momentum expansion are negligible small compared with the anomalous ones due to the inequality $M_{\rm N} \gg \Lambda_{\rm D}$ and the limit $N_C \to \infty$. This justifies the application of the approximation by the leading powers in an external momentum expansion to the evaluation of the effective Lagrangians of the deuteron coupled to itself and other fields.

Collecting all pieces we get the structure function $\Pi^{\mu\nu}(k_1;Q)$ in the form

$$\Pi^{\mu\nu}(k_1;Q) = \frac{4}{3}(k_1^2 g^{\mu\nu} - k_1^{\mu}k_1^{\nu})J_2(M_{\rm N}) + 2g^{\mu\nu}[J_1(M_{\rm N}) + M_{\rm N}^2 J_2(M_{\rm N})] - 2\,a(a+1)\,(k_1^2\,g^{\mu\nu} - 2\,k_1^{\mu}k_1^{\nu}). \quad (3.13)$$

The effective Lagrangian of the free deuteron field is then defined by

$$\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\left(-\frac{g_{\rm V}^2}{2\pi^2}\,a(a+1) + \frac{g_{\rm V}^2}{3\pi^2}\,J_2(M_{\rm N})\right)D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + \left(M_0^2 - \frac{g_{\rm V}^2}{2\pi^2}\,[J_1(M_{\rm N}) + M_{\rm N}^2 J_2(M_{\rm N})]\right)D^{\dagger}_{\mu}(x)D^{\mu}(x), \quad (3.14)$$

where $D^{\mu\nu}(x) = \partial^{\mu}D^{\nu}(x) - \partial^{\nu}D^{\mu}(x)$. We have dropped some contributions proportional to the total divergence of the deuteron field, since $\partial_{\mu}D^{\mu}(x) = 0$. For the derivation of (3.14) we have used the relation

$$\int d^4x\int\frac{d^4x_1 d^4k_1}{(2\pi)^4}\,e^{-ik_1\cdot(x - x_1)} \times D^{\dagger}_{\mu}(x)D_{\nu}(x_1)(k_1^2 g^{\mu\nu} - k_1^{\mu}k_1^{\nu}) = = \frac{1}{2}\int d^4x\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x). \quad (3.15)$$

In order to get a correct kinetic term of the free deuteron field in the effective Lagrangian (3.14) we should set

$$-\frac{g_{\rm V}^2}{2\pi^2}\,a(a+1) = 1. \quad (3.16)$$

Since a is an arbitrary real parameter, the relation (3.16) is valid in the case of the existence of real roots. For the existence of real roots of (3.16) the coupling constant $g_{\rm V}$ should obey the constraint $g_{\rm V}^2 \le 8\pi^2$ that is satisfied by the numerical value $g_{\rm V} = 11.319$ calculated at $N_C = 3$ [15]. Since $g_{\rm V} \sim O(\sqrt{N_C})$ at $N_C \to \infty$, (3.16) has real solutions for any $N_C \ge 3$.

Due to (3.16) the effective Lagrangian of the free deuteron field takes the form

$$\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\left(1 + \frac{g^2_{\rm V}}{3\pi^2}\,J_2(M_{\rm N})\right) D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + \left(M^2_0 - \frac{g^2_{\rm V}}{2\pi^2}\,[J_1(M_{\rm N}) + M^2_{\rm N}J_2(M_{\rm N})]\right) D^{\dagger}_{\mu}(x)D^{\mu}(x). \tag{3.17}$$

By performing the renormalization of the wave function of the deuteron field [14,15]

$$\left(1 + \frac{g^2_{\rm V}}{3\pi^2}\,J_2(M_{\rm N})\right)^{1/2} D^{\mu}(x) \to D^{\mu}(x) \tag{3.18}$$

and taking into account that $M_{\rm N} \gg \Lambda_{\rm D}$ we arrive at the effective Lagrangian of the free physical deuteron field

$$\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + M^2_{\rm D}\,D^{\dagger}_{\mu}(x)D^{\mu}(x), \tag{3.19}$$

where $M_{\rm D} = M_0 - \varepsilon_{\rm D}$ is the mass of the physical deuteron field. The binding energy of the deuteron $\varepsilon_{\rm D}$ reads

$$\varepsilon_{\rm D} = \frac{17}{48}\,\frac{g^2_{\rm V}}{\pi^2}\,\frac{J_1(M_{\rm N})}{M_{\rm N}} = \frac{17}{18}\,Q_{\rm D}\,\Lambda^3_{\rm D} \sim O(1/N_C). \tag{3.20}$$

We have used here the relation between divergent integrals (3.12) and expressed the phenomenological coupling constant $g_{\rm V}$ in terms of the electric quadrupole moment of the deuteron $g^2_{\rm V} = 2\pi^2 Q_{\rm D}M^2_{\rm N}$. The dependence of the physical observable parameter, the binding energy of the deuteron, on the cut–off $\Lambda_{\rm D}$ is usual for any effective theory like the NJL model [1–11].

At $N_C \to \infty$ the binding energy of the deuteron behaves like $O(1/N_C)$ as well as the electric quadrupole moment $Q_{\rm D}$ and the coupling constant of the phenomenological local four–nucleon interaction (3.1). This testifies a self–consistency of our approach. Really, all parameters of the physical deuteron field are of the same order according to the large N_C expansion. This means that the vanishing of the coupling constant of the phenomenological four–nucleon interaction (3.1) in the limit $N_C \to \infty$ entails the vanishing of all low–energy parameters of the physical deuteron.

4 Electromagnetic properties of the deuteron

The description of the deuteron as a Cooper np–pair changes the analysis of the electromagnetic parameters of the deuteron given in [15], since we do not have more a “bare” deuteron field having the magnetic dipole and electric quadrupole moment. Therefore, for the Cooper np–pair both the magnetic dipole and electric quadrupole moments have to be induced fully by the one–nucleon loop contributions. For the self–consistent description of the electromagnetic properties of the deuteron we cannot deal with only the nucleon current $j_{\mu}(x)$ given by (3.2) and have to introduce the tensor nucleon current [14,15]

$$J^{\mu\nu}(x) = \bar{p^c}(x)\sigma^{\mu\nu}n(x) - \bar{n^c}(x)\sigma^{\mu\nu}p(x). \tag{4.1}$$

The local four–nucleon interaction producing the deuteron as a Cooper np–pair reads now

$$\mathcal{L}_{\rm int}(x) = -\frac{1}{4M^2_{\rm N}}\,J^{\dagger}_{\mu}(x)J^{\mu}(x). \tag{4.2}$$

The baryon current $J^{\mu}(x)$ is defined by

$$J^{\mu}(x) = -i\,g_{\rm V}\,[\bar{p^c}(x)\gamma^{\mu}n(x) - \bar{n^c}(x)\gamma^{\mu}p(x)] - \frac{g_{\rm T}}{M_{\rm N}}\,\partial_{\nu}[\bar{p^c}(x)\sigma^{\nu\mu}n(x) - \bar{n^c}(x)\sigma^{\nu\mu}p(x)], \tag{4.3}$$

where $g_{\rm T}$ is a dimensionless phenomenological coupling constant [15]. The contribution of the tensor nucleon current looks like the next–to–leading term in the long–wavelength expansion[1] of an effective low–energy four–nucleon interaction.

The effective Lagrangian of the np–system unstable under creation of the Cooper np–pair with quantum numbers of the deuteron is then defined by

$$\mathcal{L}^{\rm np}(x) = \bar{n}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})n(x) + \bar{p}(x)(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})p(x) - \frac{1}{4M^2_{\rm N}}J^{\dagger}_{\mu}(x)J^{\mu}(x). \tag{4.4}$$

The linearalized version of the effective Lagrangian (4.4) containing the interpolating local deuteron field reads

$$\mathcal{L}^{\rm np}(x) \to \bar{n}(x)(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})n(x) + \bar{p}(x)(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})p(x) + M^2_0 D^{\dagger}_{\mu}(x)D^{\mu}(x) + g_{\rm V}j^{\dagger}_{\mu}(x)D^{\mu}(x) + g_{\rm V}j^{\mu}(x)D^{\dagger}_{\mu}(x) + \frac{g_{\rm T}}{M_0}J^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + \frac{g_{\rm T}}{M_0}J^{\mu\nu}(x)D^{\dagger}_{\mu\nu}(x), \tag{4.5}$$

where $M_0 = 2M_{\rm N}$, $D^{\mu}(x)$ is a local interpolating field with quantum numbers of the deuteron and $D^{\mu\nu}(x) = \partial^{\mu}D^{\nu}(x) - \partial^{\nu}D^{\mu}(x)$.

The interactions with the tensor current give the contributions only to the divergent part of the effective Lagrangian of the free deuteron field determined now by

$$\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\left(-\frac{g^2_{\rm V}}{2\pi^2}a(a+1) + \frac{g^2_{\rm V} + 6g_{\rm V}g_{\rm T} + 3g^2_{\rm T}}{3\pi^2}J_2(M_{\rm N})\right) \times D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + \left(M^2_0 - \frac{g^2_{\rm V}}{2\pi^2}[J_1(M_{\rm N}) + M^2_{\rm N}J_2(M_{\rm N})]\right) \times D^{\dagger}_{\mu}(x)D^{\mu}(x). \tag{4.6}$$

The one–nucleon loop diagrams defining the effective Lagrangian (4.6) are depicted in Fig. 1.

[1] Due to proportionality $M_{\rm N} \sim N_C$ this expansion is related to the large N_C expansion.

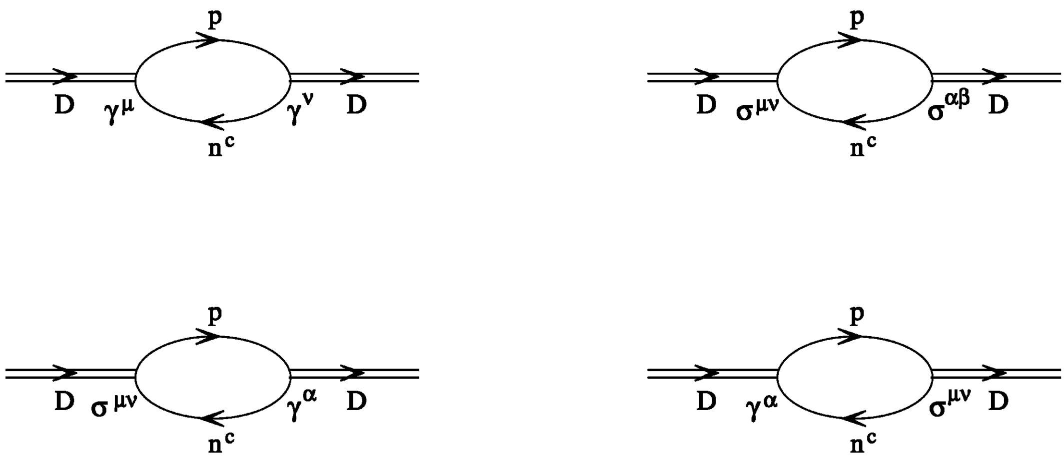

Fig. 1. One–nucleon loop diagrams contributing in the NNJL model to the binding energy of the physical deuteron, where $n^c = C\,\bar{n}^T$ is the field of anti–nucleon

Due to the relation (3.16) the effective Lagrangian of the free deuteron field (4.6) takes the form

$$\begin{aligned}&\mathcal{L}_{\rm eff}(x) = \\ &-\frac{1}{2}\Bigg(1 + \frac{g^2_{\rm V} + 6 g_{\rm V} g_{\rm T} + 3 g^2_{\rm T}}{3\pi^2} J_2(M_{\rm N})\Bigg) \\ &\times D^{\dagger}_{\mu\nu}(x) D^{\mu\nu}(x) \\ &+\Bigg(M^2_0 - \frac{g^2_{\rm V}}{2\pi^2}[J_1(M_{\rm N}) + M^2_{\rm N} J_2(M_{\rm N})]\Bigg) \\ &\times D^{\dagger}_{\mu}(x) D^{\mu}(x). \end{aligned} \tag{4.7}$$

After the renormalization of the wave function of the deuteron field we arrive at the effective Lagrangian defined by (3.19) with the binding energy of the deuteron depending on $g_{\rm V}$ and $g_{\rm T}$

$$\begin{aligned}\varepsilon_{\rm D} &= \frac{17}{48}\frac{g^2_{\rm V}}{\pi^2}\frac{J_1(M_{\rm N})}{M_{\rm N}}\Bigg(1 + \frac{48}{17}\frac{g_{\rm T}}{g_{\rm V}} + \frac{24}{17}\frac{g^2_{\rm T}}{g^2_{\rm V}}\Bigg) \\ &= \frac{17}{18}\, Q_{\rm D}\, \Lambda^3_{\rm D}\Bigg(1 + \frac{48}{17}\frac{g_{\rm T}}{g_{\rm V}} + \frac{24}{17}\frac{g^2_{\rm T}}{g^2_{\rm V}}\Bigg), \end{aligned} \tag{4.8}$$

where we have used the relation between divergent integrals (3.12) and expressed the phenomenological coupling constant $g_{\rm V}$ in terms of the electric quadrupole moment of the deuteron $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$. In order to make the prediction for the binding energy much more definite we have to know the relation between the phenomenological coupling constants $g_{\rm V}$ and $g_{\rm T}$. For this aim we suggest to consider the electromagnetic properties of the deuteron.

Including the electromagnetic field by a minimal way $\partial_\mu \to \partial_\mu + i\, e\, A_\mu(x)$, where e and $A_\mu(x)$ are the electric charge of the proton and the electromagnetic potential we bring up the linearalized version of the Lagrangian (4.5) to the form

$$\begin{aligned}&\mathcal{L}^{\rm np}(x) \to \mathcal{L}^{\rm np}_{\rm ELM}(x) \\ &= \bar{n}(x)(i\gamma^\mu\partial_\mu - M_{\rm N})n(x) + \bar{p}(x)(i\gamma^\mu\partial_\mu - M_{\rm N})p(x) \\ &+ M^2_0 D^{\dagger}_{\mu}(x) D^{\mu}(x) + g_{\rm V} j^{\dagger}_{\mu}(x) D^{\mu}(x) + g_{\rm V} j^{\mu}(x) D^{\dagger}_{\mu}(x) \\ &+ \frac{g_{\rm T}}{M_0} J^{\dagger}_{\mu\nu} D^{\mu\nu}(x) + \frac{g_{\rm T}}{M_0} J^{\mu\nu} D^{\dagger}_{\mu\nu}(x) - e\,\bar{p}(x)\gamma^\mu p(x)\, A_\mu(x) \\ &- i\, e\,\frac{g_{\rm T}}{M_0} J^{\dagger}_{\mu\nu}(x)(A^\mu(x) D^\nu(x) - A^\nu(x) D^\mu(x)) \\ &+ i\, e\,\frac{g_{\rm T}}{M_0} J^{\mu\nu}(x)(A_\mu(x) D_\nu(x) - A^\nu(x) D^\mu(x)). \end{aligned} \tag{4.9}$$

By using this Lagrangian we should calculate fully all contributions to the effective Lagrangian of the deuteron coupled to an external electromagnetic field. These are the effective Lagrangians of the Corben–Schwinger [18] and the Aronson [19] type defining at a field theoretic level the magnetic dipole and the electric quadrupole moment of the deuteron, and the effective interactions which can be identified with the contributions caused by the minimal inclusion of the electromagnetic field $\partial_\mu D_\nu(x) \to (\partial_\mu + i\, e\, A_\mu(x)) D_\nu(x)$.

4.1 The phenomenological Corben–Schwinger interaction

The one–nucleon loop diagrams defining in the NNJL model effective electromagnetic interactions of the deuteron linear in electric charge e induced by the Lagrangian (4.9) are depicted in Fig. 2. One can show that in the $1/M_{\rm N}$ expansion corresponding to the large N_C expansion due to the proportionality $M_{\rm N} \sim N_C$ [17] the one–nucleon loop diagrams in Fig. 2a and 2b are divergent. Therefore, due to (3.12) at leading order in the large N_C expansion the contributions of these diagrams can be neglected with respect to the contributions of the diagrams in Fig. 2c and 2d defining the phenomenological Lagrangians of the Corben–Schwinger, $\mathcal{L}_{\rm CS}(x)$, and the Aronson, $\mathcal{L}_{\rm A}(x)$, type, respectively, in terms of the nucleon–loop anomalies [15].

The effective Lagrangian of the diagram in Fig. 2c is defined by [15]

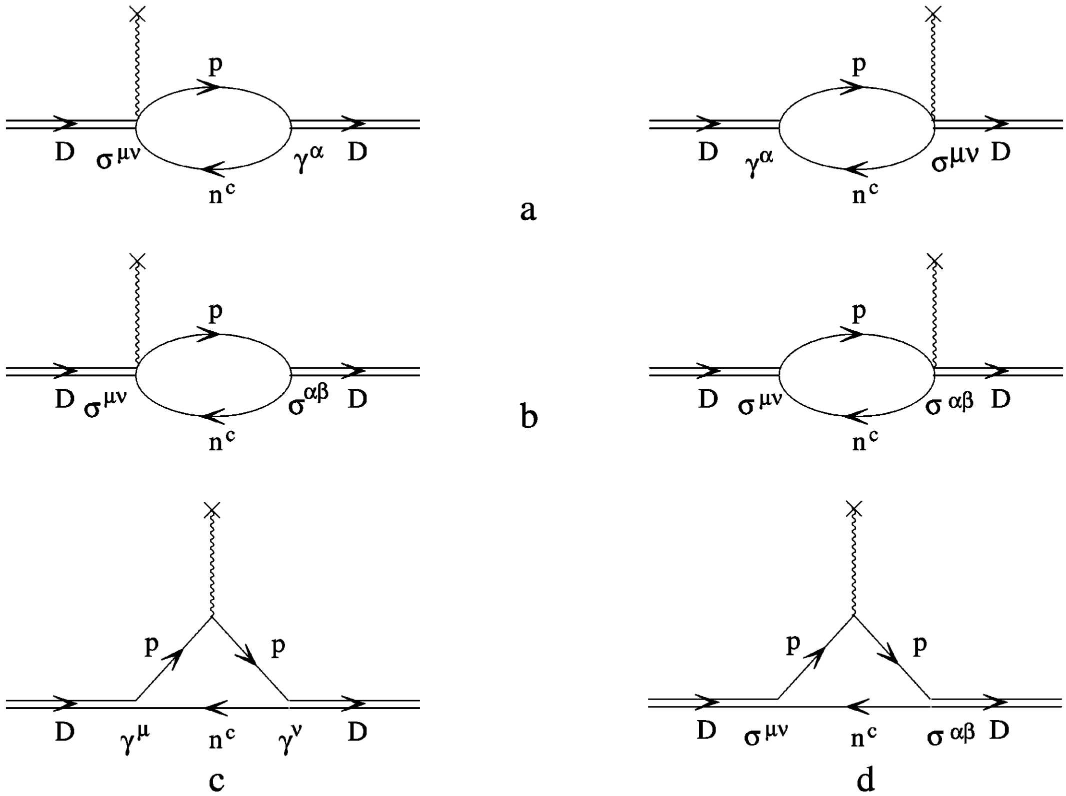

Fig. 2. One–nucleon loop diagrams describing in the NNJL model the effective Lagrangian of the deuteron coupled to an electromagnetic field through the magnetic dipole and electric quadrupole moments, where $n^c = C\,\bar{n}^T$ is the field of anti–nucleon

$$\int d^4x\,\mathcal{L}_{\rm Fig.\ 2c}(x) =$$
$$\int d^4x \int \frac{d^4x_1 d^4k_1}{(2\pi)^4}\,\frac{d^4x_2 d^4k_2}{(2\pi)^4}\,D_{\beta}(x)\,D^{\dagger}_{\alpha}(x_1)\,A_{\mu}(x_2)$$
$$\times\, e^{-i\,k_1\cdot x_1}\,e^{-i\,k_2\cdot x_2}\,e^{i\,(k_1+k_2)\cdot x}\,\frac{eg^2_{\rm V}}{4\pi^2}\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q). \tag{4.10}$$

In the one–nucleon loop approximation the structure function $\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q)$ is given by the momentum integral [15]

$$\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = \int\frac{d^4\,k}{\pi^2\,i}$$
$$\times\,{\rm tr}\Big\{\gamma^{\beta}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}}\gamma^{\alpha}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1}$$
$$\cdot\,\gamma^{\mu}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1-\hat{k}_2}\Big\}. \tag{4.11}$$

The 4–vector $Q = a\,k_1 + b\,k_2$, where a and b are arbitrary parameters, displays the dependence of the k integral in (4.11) on a shift of a virtual momentum. According to [29,31] a Q–dependent part of an one–nucleon loop diagram is related to the anomaly of this diagram. Therefore, the evaluation of the Q–dependence of the one–nucleon loop diagram should play an important role in the NNJL model. For the evaluation of the Q–dependence of the structure function (4.11) we apply the method invented by Gertsein and Jackiw [29] and consider the following difference of momentum integrals

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q)\;=\;\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q\,)-\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;0) \tag{4.12}$$

In accordance with the Gertsein–Jackiw method the difference (4.12) can be represented by the integral

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q)\;=\;\int^1_0 dx\,\frac{d}{d\,x}\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;x\,Q)$$
$$=\int^1_0 d\,x\int\frac{d^4\,k}{\pi^2\,i}\,Q^{\lambda}\,\frac{\partial}{\partial\,k^{\lambda}}$$
$$\times\,{\rm tr}\Big\{\gamma^{\beta}\,\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}}\,\gamma^{\alpha}\,\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}-\hat{k}_1}\,\gamma^{\mu}$$
$$\times\,\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}-\hat{k}_1-\hat{k}_2}\Big\}. \tag{4.13}$$

This shows that the contribution of the Q–dependent part of the structure function (4.11) is just the surface term. Following Gertsein and Jackiw [29] and evaluating the integral over k symmetrically we obtain

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = -\,2\int^1_0 dx\,\lim_{k\to\infty}\Big\langle\frac{Q\cdot k}{k^4}\mathrm{tr}\{\gamma^\beta\,(M_{\rm N}+\hat{k}+x\hat{Q})\,\gamma^\alpha\,\times \times(M_{\rm N}+\hat{k}+x\hat{Q}+\hat{k}_1)\,\gamma^\mu\,(M_{\rm N}+\hat{k}+x\hat{Q}+\hat{k}_1+\hat{k}_2)\}\Big\rangle. \tag{4.14}$$

The brackets $< \ldots >$ mean the averaging over k directions. Due to the limit $k\to\infty$ we can neglect all momenta with respect to k.

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) \,=\, -\,2\lim_{R\to\infty}\Big\langle\frac{Q\cdot k}{k^4}\mathrm{tr}\{\gamma^\beta\hat{k}\gamma^\alpha\hat{k}\gamma^\mu\hat{k}\}\Big\rangle. \tag{4.15}$$

Averaging over k–directions

$$\lim_{k\to\infty}\frac{k^\lambda k^\varphi k^\omega k^\rho}{k^4} \,=\, \frac{1}{24}\,(g^{\lambda\varphi}g^{\omega\rho}+g^{\lambda\omega}g^{\varphi\rho}+g^{\lambda\rho}g^{\varphi\omega}) \tag{4.16}$$

we obtain

$$\begin{aligned}\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) &= -\,\frac{1}{12}\,\mathrm{tr}(\gamma_\lambda\gamma^\beta\gamma^\lambda\gamma^\alpha\hat{Q}\gamma^\mu \\ &\quad +\gamma^\beta\gamma_\lambda\gamma^\alpha\gamma^\lambda\hat{Q}\gamma^\mu+\gamma_\beta\hat{Q}\gamma^\alpha\gamma_\lambda\gamma^\mu\gamma^\lambda) \\ &= \frac{2}{3}\,(Q^\alpha g^{\beta\mu}+Q^\beta g^{\mu\alpha}+Q^\mu g^{\alpha\beta}).\end{aligned} \tag{4.17}$$

Our result (4.17) agrees with the statement by Gertsein and Jackiw [29] that the Q–dependence of one–nucleon loop diagrams, i.e. the anomaly of the one–nucleon loop diagram, is fully defined by the surface behavior of the integrand of the momentum integral at a virtual momentum going to infinity, $k\to\infty$. This relates the anomalies of the one–nucleon loop diagrams with contributions of high–energy (short–distance) fluctuations of virtual nucleon and anti–nucleon fields, i.e. the $N\bar{N}$ fluctuations.

Now we can proceed to the evaluation of $\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q)$. In order to pick up the contribution of the Q–dependent part one cannot apply the Feynman method of the evaluation of momentum integrals like (4.11). This method involves the mergence of the factors in the denominator with the subsequent shift of a virtual momentum. On this way one can lose the Q–dependence by virtue of the shift at the intermediate stage. Thereby, we have to evaluate the integral over k without any intermediate shifts.

One can make this by applying a momentum expansion related to the $1/M_{\rm N}$ expansion or that is the same the large N_C expansion [17] and keeping only the leading terms.

$$\begin{aligned}&\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) \,= \\ &= \int\frac{d^4\,k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^\beta\,\frac{M_{\rm N}+\hat{k}+\hat{Q}}{M^2_{\rm N}-k^2}\,\Big[1+\frac{2\,k\cdot Q}{M^2_{\rm N}-k^2}\Big] \\ &\times\gamma^\alpha\,\frac{M_{\rm N}+\hat{k}+\hat{Q}+\hat{k}_1}{M^2_{\rm N}-k^2}\,\times \\ &\times\Big[1+\frac{2\,k\cdot(Q+k_1)}{M^2_{\rm N}-k^2}\Big]\,\gamma^\mu\,\frac{M_{\rm N}+\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2}{M^2_{\rm N}-k^2} \\ &\times\Big[1+\frac{2\,k\cdot(Q+k_1+k_2)}{M^2_{\rm N}-k^2}\Big]\Big\} = \\ &= \int\frac{d^4\,k}{\pi^2\,i}\,\frac{1}{(M^2_{\rm N}-k^2)^3}\,\mathrm{tr}\,\{M^2_{\rm N}\gamma^\beta(\hat{k}+\hat{Q})\gamma^\alpha\gamma^\mu \\ &+M^2_{\rm N}\gamma^\beta\gamma^\alpha(\hat{k}+\hat{Q}+\hat{k}_1)\gamma^\mu \\ &+M^2_{\rm N}\gamma^\beta\gamma^\alpha\gamma^\mu(\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2) \\ &+\gamma^\beta(\hat{k}+\hat{Q})\gamma^\alpha(\hat{k}+\hat{Q}+\hat{k}_1) \\ &\times\gamma^\mu(\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2)\}\Big[1+\frac{2\,k\cdot(3Q+2k_1+k_2)}{M^2_{\rm N}-k^2}\Big] = \\ &= \frac{1}{2}\int\frac{d^4\,k}{\pi^2\,i}\Big[\frac{1}{(M^2_{\rm N}-k^2)^2}+\frac{M^2_{\rm N}}{(M^2_{\rm N}-k^2)^3}\Big] \\ &\times\mathrm{tr}\,\{\gamma^\beta\hat{Q}\gamma^\alpha\gamma^\mu+\gamma^\beta\gamma^\alpha(\hat{Q}+\hat{k}_1)\gamma^\mu \\ &+\gamma^\beta\gamma^\alpha\gamma^\mu(\hat{Q}+\hat{k}_1+\hat{k}_2)\} \\ &+2\int\frac{d^4\,k}{\pi^2\,i}\frac{k\cdot(3Q+2k_1+k_2)}{(M^2_{\rm N}-k^2)^3} \\ &\times\mathrm{tr}\,\{M^2_{\rm N}(\gamma^\beta\hat{k}\gamma^\alpha\gamma^\mu+\gamma^\beta\gamma^\alpha\hat{k}\gamma^\mu+\gamma^\beta\gamma^\alpha\gamma^\mu\hat{k}) \\ &+\gamma^\beta\hat{k}\gamma^\alpha\hat{k}\gamma^\mu\hat{k}\} \,=\, \mathcal{J}^{\beta\alpha\mu}_{(1)}(k_1,k_2;Q)+\mathcal{J}^{\beta\alpha\mu}_{(2)}(k_1,k_2;Q).\end{aligned} \tag{4.18}$$

For the evaluation of $\mathcal{J}^{\beta\alpha\mu}_{(1)}(k_1,k_2;Q)$ it is sufficient to calculate the trace of the Dirac matrices and integrate over k

$$\begin{aligned}&\mathcal{J}^{\beta\alpha\mu}_{(1)}(k_1,k_2;Q) \,=\, [1+2\,J_2(M_{\rm N})]\,[(Q+2\,k_1+k_2)^\alpha g^{\beta\,\mu} \\ &+(Q+2\,k_1+k_2)^\beta g^{\mu\,\alpha}+(Q+2\,k_1+k_2)^\mu g^{\alpha\,\beta} \\ &-2\,(k_1+k_2)^\alpha\,g^{\beta\,\mu}-2\,k^\beta_1\,g^{\mu\,\alpha}],\end{aligned} \tag{4.19}$$

where $J_2(M_{\rm N})$ describes a divergent contribution depending on the cut–off $\Lambda_{\rm D}$. Due to inequality $M_{\rm N}\gg\Lambda_{\rm D}$ we can neglect $J_2(M_{\rm N})$ with respect to the convergent contribution. This corresponds too the accounting for the leading contributions in the large N_C expansion. Indeed, according to (3.12) the contribution of divergent integrals is of order $O(1/N_C)$ relative to the convergent ones.

For the evaluation of $\mathcal{J}^{\beta\alpha\mu}_{(2)}(k_1,k_2;Q)$ it is convenient, first, to integrate over k directions and then to calculate the trace over Dirac matrices. This gives

$$\begin{aligned}&\mathcal{J}^{\beta\alpha\mu}_{(2)}(k_1,k_2;Q) \,= \\ &=\int\frac{d^4\,k}{\pi^2\,i}\Big[\frac{1}{2}\frac{M^2_{\rm N}k^2}{(M^2_{\rm N}-k^2)^4}-\frac{1}{6}\frac{k^4}{(M^2_{\rm N}-k^2)^4}\Big](3Q+k_1+k_2)_\lambda \\ &\qquad\times\mathrm{tr}\,(\gamma^\beta\gamma^\lambda\gamma^\alpha\gamma^\mu+\gamma^\beta\gamma^\alpha\gamma^\lambda\gamma^\mu+\gamma^\beta\gamma^\alpha\gamma^\mu\gamma^\lambda) \,= \\ &= -\,\frac{1}{9}[1+6\,J_2(M_{\rm N})]\,[(3\,Q+2\,k_1+k_2)^\alpha g^{\beta\,\mu} \\ &+(3\,Q+2\,k_1+k_2)^\beta g^{\mu\,\alpha}+(3\,Q+2\,k_1+k_2)^\mu g^{\alpha\,\beta}].\end{aligned} \tag{4.20}$$

Here we have used the integrals

$$\int \frac{d^4k}{\pi^2\, i}\,\frac{1}{(M^2_{\rm N} - k^2)^3} = \frac{1}{2\,M^2_{\rm N}},$$
$$\int \frac{d^4k}{\pi^2\, i}\,\frac{1}{(M^2_{\rm N} - k^2)^4} = \frac{1}{6\,M^4_{\rm N}}. \qquad (4.21)$$

Summing up the contributions we obtain

$$\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) \;=\; \frac{2}{3}\,(Q^{\alpha}\,g^{\beta\,\mu} + Q^{\beta}\,g^{\mu\,\alpha} + Q^{\mu}\,g^{\alpha\,\beta})$$
$$+\,\frac{8}{9}\,[1 + \frac{3}{2}\,J_2(M_{\rm N})]$$
$$\times[(2\,k_1 + k_2)^{\alpha}\,g^{\beta\,\mu} + (2\,k_1 + k_2)^{\beta}\,g^{\mu\,\alpha} + (2\,k_1 + k_2)^{\mu}\,g^{\alpha\,\beta}]$$
$$+\,[1 + 2\,J_2(M_{\rm N})][-\,2\,(k_1 + k_2)^{\alpha}\,g^{\beta\mu} - 2\,k^{\beta}_1\,g^{\mu\alpha}]. \qquad (4.22)$$

It is seen that the Q–dependence coincides with that obtained by means of the Gertsein–Jackiw method (4.17). Due to the arbitrariness of Q we can absorb by the Q–term the terms having the same Lorentz structure. This brings up the r.h.s. of (4.22) to the form

$$\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = \frac{2}{3}\,(Q^{\alpha}g^{\beta\mu} + Q^{\beta}g^{\mu\alpha} + Q^{\mu}g^{\alpha\,\beta}) + [-\,2\,(k_1 + k_2)^{\alpha}g^{\beta\mu} - 2\,k^{\beta}_1 g^{\mu\alpha}]. \qquad (4.23)$$

Also we have dropped here the divergent contribution. This approximation is valid due to the inequality $M_{\rm N} \gg \Lambda_{\rm D}$ and at leading order in the large N_C expansion.

The effective Lagrangian $\mathcal{L}_{\rm Fig.\ 2c}(x)$ determined by the structure function (4.23) reads

$$\mathcal{L}_{\rm Fig.\ 2c}(x) = i\,e\,\frac{g^2_{\rm V}}{6\pi^2}[(3 - a)\,\partial^{\mu}D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,A^{\nu}(x)$$
$$-\,(3 - a)\,D^{\dagger}_{\mu}(x)\,\partial^{\nu}D_{\nu}(x)\,A^{\mu}(x) - b\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,\partial^{\mu}\,A^{\nu}(x)$$
$$-\,(b - a)\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,\partial^{\nu}\,A^{\mu}(x)$$
$$-\,(a - b)\,\partial^{\nu}\,D^{\dagger}_{\mu}(x)\,D^{\mu}(x)\,A_{\nu}(x) + b\,D^{\dagger}_{\mu}(x)\,\partial^{\nu}\,D^{\mu}(x)\,A_{\nu}(x)$$
$$+\,3\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,(\partial^{\mu}\,A^{\nu}(x) - \partial^{\nu}\,A^{\mu}(x))]. \qquad (4.24)$$

Due to the constraints $\partial^{\mu}D^{\dagger}_{\mu}(x) = \partial^{\mu}D_{\mu}(x) = 0$ some terms in the Lagrangian (4.24) can be dropped out. This gives

$$\mathcal{L}_{\rm Fig.\ 2c}(x) =$$
$$= i\,e\,\frac{g^2_{\rm V}}{6\pi^2}[-\,b\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,\partial^{\mu}A^{\nu}(x)$$
$$-\,(b - a)\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,\partial^{\nu}A^{\mu}(x)$$
$$-\,(a - b)\,\partial^{\nu}D^{\dagger}_{\mu}(x)\,D^{\mu}(x)\,A_{\nu}(x) + b\,D^{\dagger}_{\mu}(x)\,\partial_{\nu}D^{\mu}(x)\,A^{\nu}(x)$$
$$+\,3\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,(\partial^{\mu}A^{\nu}(x) \,-\, \partial^{\nu}A^{\mu}(x))]. \qquad (4.25)$$

By using the relations $\partial_{\nu}D^{\dagger}_{\mu}(x) = D^{\dagger}_{\nu\mu}(x) + \partial_{\mu}D^{\dagger}_{\nu}(x)$ and $\partial_{\nu}D_{\mu}(x) = D_{\nu\mu}(x) + \partial_{\mu}D_{\nu}(x)$ we can rewrite the Lagrangian (4.25) as follows

$$\mathcal{L}_{\rm Fig.\ 2c}(x) =$$
$$= i\,e\,\frac{g^2_{\rm V}}{6\pi^2}[-\,(a - b)\,D^{\dagger}_{\nu\mu}(x)\,A^{\nu}(x)\,D^{\mu}(x)$$
$$+b\,D^{\nu\mu}(x)\,A_{\nu}(x)\,D^{\dagger}_{\mu}(x)$$
$$-\,(a - b)\,\partial_{\nu}D^{\dagger}_{\mu}(x)\,D^{\nu}(x)\,A^{\mu}(x) + b\,D^{\dagger}_{\mu}(x)\,\partial^{\mu}D_{\nu}(x)\,A^{\nu}(x)$$
$$-\,b\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,\partial^{\mu}A^{\nu}(x) - (b - a)\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,\partial^{\nu}A^{\mu}(x)$$
$$+\,3\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,(\partial^{\mu}A^{\nu}(x) \,-\, \partial^{\nu}A^{\mu}(x))]. \qquad (4.26)$$

The subsequent transformations we perform by applying the identity

$$\partial^{\nu}D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,A^{\mu}(x) \,-\, D^{\dagger}_{\mu}(x)\,\partial^{\mu}D_{\nu}(x)\,A^{\nu}(x) =$$
$$= D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,(\partial^{\mu}A^{\nu}(x) - \partial^{\nu}\,A^{\mu}(x)) \qquad (4.27)$$

being valid up to the contribution of a total divergence which can be omitted. Setting $a = 2\,b$ we represent the effective Lagrangian (4.27) in the irreducible form

$$\mathcal{L}_{\rm Fig.\ 2c}(x) = i\,e\,\frac{g^2_{\rm V}}{6\pi^2}[b\,D^{\dagger}_{\mu\nu}(x)\,A^{\nu}(x)\,D^{\mu}(x) - b\,D^{\mu\nu}(x)\,A_{\nu}(x)\,D^{\dagger}_{\mu}(x)] + i\,e\,\frac{g^2_{\rm V}}{6\pi^2}(3 - 2b)\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,F^{\mu\nu}(x), \qquad (4.28)$$

where $F^{\mu\nu}(x) = \partial^{\mu}A^{\nu}(x) - \partial^{\nu}A^{\mu}(x)$ is the electromagnetic field strength tensor. Then, first two terms define the finite contributions to the renormalization constant of the wave function of the deuteron, whereas the last term coincides with the well–known phenomenological interaction $\mathcal{L}_{\rm CS}(x)$ introduced by Corben and Schwinger [18]

$$\mathcal{L}_{\rm CS}(x) = i\,e\,\frac{g^2_{\rm V}}{6\pi^2}\,(3 - 2b)\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,F^{\mu\nu}(x) \qquad (4.29)$$

for the description of the charged vector field coupled to an external electromagnetic field.

Thus, at leading order in the large N_C expansion the anomaly of the one–nucleon loop triangle VVV–diagram with vector (V) vertices defines fully the effective Lagrangian of the Corben–Schwinger type describing the deuteron coupled to an external electromagnetic field. In turn, the finite the contributions to the renormalization constant of the wave function of the deuteron we will identify below with those induced by a minimal inclusion of the electromagnetic interaction: $\partial_{\mu}D_{\nu}(x) \to (\partial_{\mu} + i\,e\,A_{\nu}(x))D_{\nu}(x)$. These terms are important for the correct definition of the effective Lagrangian of the deuteron coupled to an electromagnetic field.

4.2 The phenomenological Aronson interaction

The effective Lagrangian described by the diagram in Fig. 2d is defined by [15]

$$\int d^4x\,\mathcal{L}_{\rm Fig.\ 2d}(x) =$$
$$\int d^4x \int \frac{d^4x_1\,d^4k_1}{(2\pi)^4}\,\frac{d^4x_2\,d^4k_2}{(2\,\pi)^4}\,D_{\alpha\,\beta}(x)\,D^{\dagger}_{\mu\,\nu}(x_1)\,A_{\lambda}(x_2)$$
$$\times\, e^{-\,i\,k_1\cdot x_1}\,e^{-\,i\,k_2\cdot x_2}\,e^{i\,(k_1\,+\,k_2)\cdot x}\,(-\,e)\,\frac{g^2_{\rm T}}{4\pi^2}$$
$$\frac{1}{M^2_{\rm D}}\,\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q). \qquad (4.30)$$

In the structure function $\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q)$ is represented by the following momentum integral

$$\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) \,=\, \int \frac{d^4k}{\pi^2\,i}$$
$$\times{\rm tr}\Bigg\{\sigma^{\alpha\,\beta}\,\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}}\,\sigma^{\mu\,\nu}\,\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1}$$
$$\times\,\gamma^{\lambda}\,\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1-\hat{k}_2}\Bigg\}. \qquad (4.31)$$

The 4–vector $Q = a\,k_1 + b\,k_2$ is an arbitrary shift of a virtual momentum, where a and b are arbitrary parameters. The Q–dependent part of the structure function we obtain by using the Gertsein–Jackiw method [15]

$$\delta\,\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) =$$
$$-\,\frac{1}{12}\,{\rm tr}(\gamma_{\rho}\,\sigma^{\alpha\,\beta}\,\gamma^{\rho}\,\sigma^{\mu\,\nu}\,\hat{Q}\,\gamma^{\lambda}\,+\,\sigma^{\alpha\,\beta}\,\gamma_{\rho}\,\sigma^{\mu\,\nu}\,\gamma^{\rho}\,\hat{Q}\,\gamma^{\lambda}\,+$$
$$+\,\sigma^{\alpha\,\beta}\,\hat{Q}\,\sigma^{\mu\,\nu}\,\gamma_{\rho}\,\gamma^{\lambda}\,\gamma^{\rho}) = \frac{1}{6}\,{\rm tr}(\sigma^{\alpha\,\beta}\,\hat{Q}\,\sigma^{\mu\,\nu}\,\gamma^{\lambda}\,). \qquad (4.32)$$

Now we should proceed to the evaluation of $\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q)$. By analogy with $\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q)$ we get

$$\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) \,=$$
$$=\int\frac{d^4k}{\pi^2\,i}\,{\rm tr}\Bigg\{\sigma^{\alpha\,\beta}\,\frac{M_{\rm N}+\hat{k}+\hat{Q}}{M^2_{\rm N}-k^2}\left[1\,+\,\frac{2\,k\cdot Q}{M^2_{\rm N}-k^2}\right]$$
$$\times\sigma^{\mu\,\nu}\,\frac{M_{\rm N}+\hat{k}+\hat{Q}+\hat{k}_1}{M^2_{\rm N}-k^2}$$
$$\times\left[1\,+\,\frac{2\,k\cdot(Q+k_1)}{M^2_{\rm N}-k^2}\right]\gamma^{\lambda}\,\frac{M_{\rm N}+\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2}{M^2_{\rm N}-k^2}$$
$$\times\left[1\,+\,\frac{2\,k\cdot(Q+k_1+k_2)}{M^2_{\rm N}-k^2}\right]\Bigg\}\,=$$
$$=\int\frac{d^4k}{\pi^2 i}\,\frac{1}{(M^2_{\rm N}-k^2)^3}{\rm tr}\,\{M^2_{\rm N}[\sigma^{\alpha\,\beta}(\hat{k}+\hat{Q})\sigma^{\mu\,\nu}\gamma^{\lambda}$$
$$+\,\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}(\hat{k}+\hat{Q}+\hat{k}_1)\gamma^{\lambda}$$
$$+\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}\gamma^{\lambda}(\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2)]$$
$$+\sigma^{\alpha\,\beta}(\hat{k}+\hat{Q})\sigma^{\mu\,\nu}(\hat{k}+\hat{Q}+\hat{k}_1)\gamma^{\lambda}$$
$$\times(\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2)\}\left[1\,+\,\frac{2\,k\cdot(3Q+2k_1+k_2)}{M^2_{\rm N}-k^2}\right]\,=$$
$$=\int\frac{d^4k}{\pi^2 i}\,\frac{1}{(M^2_{\rm N}-k^2)^3}{\rm tr}\,\{M^2_{\rm N}[\sigma^{\alpha\,\beta}\hat{Q}\sigma^{\mu\,\nu}\gamma^{\lambda}$$
$$+\,\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}(\hat{Q}+\hat{k}_1)\gamma^{\lambda}$$
$$+\,\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}\gamma^{\lambda}(\hat{Q}+\hat{k}_1+\hat{k}_2)]-\frac{1}{2}k^2\sigma^{\alpha\,\beta}\hat{Q}\sigma^{\mu\,\nu}\gamma^{\lambda}\}$$
$$+\,2\int\frac{d^4k}{\pi^2 i}\,\frac{1}{(M^2_{\rm N}-k^2)^4}$$
$$\times\,{\rm tr}\,\{\frac{1}{2}M^2_{\rm N}k^2[\sigma^{\alpha\,\beta}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)\sigma^{\mu\,\nu}\gamma^{\lambda}$$
$$+\,\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)\gamma^{\lambda}$$
$$+\,\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}\gamma^{\lambda}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)]$$
$$-\,\frac{1}{6}\,k^4\sigma^{\alpha\,\beta}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)\sigma^{\mu\,\nu}\gamma^{\lambda}\}\,=$$
$$=\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(1)}(k_1,k_2;Q)\,+\,\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(2)}(k_1,k_2;Q). \qquad (4.33)$$

Integrating over k we obtain

$$\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(1)}(k_1,k_2;Q)\,=\,\frac{1}{4}\,[1+2\,J_2(M_{\rm N})]\,{\rm tr}(\sigma^{\alpha\,\beta}\hat{Q}\sigma^{\mu\,\nu}\gamma^{\lambda})$$
$$+\frac{1}{2}\,{\rm tr}\,[\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}(\hat{Q}+\hat{k}_1)\gamma^{\lambda}+\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}\gamma^{\lambda}(\hat{Q}+\hat{k}_1+\hat{k}_2)],$$
$$\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(2)}(k_1,k_2;Q)\,=\,-\frac{1}{6}\,[-\frac{5}{6}+J_2(M_{\rm N})]$$
$$\times\,{\rm tr}[\sigma^{\alpha\,\beta}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)\sigma^{\mu\,\nu}\gamma^{\lambda}]$$
$$-\frac{1}{6}\,{\rm tr}\,[\sigma^{\alpha\,\beta}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)\sigma^{\mu\,\nu}\gamma^{\lambda}$$
$$+\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)\gamma^{\lambda}$$
$$+\sigma^{\alpha\,\beta}\sigma^{\mu\nu}\gamma^{\lambda}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)]. \qquad (4.34)$$

Now we should sum up the contributions and collect like terms

$$\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q)=\frac{1}{6}\,{\rm tr}[\sigma^{\alpha\,\beta}(\hat{Q}-\frac{1}{6}(2\,\hat{k}_1+\hat{k}_2))\sigma^{\mu\,\nu}\gamma^{\lambda}]$$
$$+\frac{1}{6}\,{\rm tr}\,[\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}(\hat{k}_1-\hat{k}_2)\gamma^{\lambda}]$$
$$+\,\frac{1}{6}\,{\rm tr}\,[\sigma^{\alpha\,\beta}\sigma^{\mu\,\nu}\gamma^{\lambda}(\hat{k}_1\,+\,2\,\hat{k}_2)]. \qquad (4.35)$$

It is seen that the Q–dependence agrees with that obtained by the Gertsein–Jackiw method. Due to arbitrariness of Q the vector $(2\,\hat{k}_1+\hat{k}_2)/6$ can be removed by the redefinition of Q. This gives

$$\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q)=\frac{1}{6}\,{\rm tr}(\sigma^{\alpha\beta}\hat{Q}\sigma^{\mu\nu}\gamma^{\lambda})$$
$$+\frac{1}{6}\,{\rm tr}\,[\sigma^{\alpha\beta}\sigma^{\mu\nu}(\hat{k}_1-\hat{k}_2)\gamma^{\lambda}]$$
$$+\frac{1}{6}\,{\rm tr}\,[\sigma^{\alpha\beta}\sigma^{\mu\,\nu}\gamma^{\lambda}(\hat{k}_1\,+\,2\,\hat{k}_2)]. \qquad (4.36)$$

By evaluating the traces over Dirac matrices we obtain the structure function leading to the following effective Lagrangian [15]

$$\mathcal{L}_{\rm Fig.\ 2d}(x) = (-\,i\,e)\,\frac{g^2_{\rm T}}{4\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,\Big[\frac{8}{3}\,a\,\partial_{\lambda}\,D^{\dagger\,\lambda\,\nu}(x)\,D_{\nu\,\mu}(x)\,A^{\mu}(x) + \frac{8}{3}\,a\,D^{\dagger\,\mu\,\nu}(x)\,\partial_{\lambda}\,D^{\lambda\,\nu}\,A_{\mu}(x) + \frac{8}{3}\,(b\,+\,a)\,D^{\dagger}_{\mu\,\nu}(x)\,D^{\nu\,\lambda}(x)\,\partial^{\mu}\,A_{\lambda}(x) + \frac{8}{3}\,(b-a)\,D^{\dagger}_{\mu\,\nu}(x)\,D^{\nu\,\lambda}(x)\,\partial_{\lambda}\,A^{\mu}(x) - \frac{16}{3}\,D^{\dagger}_{\mu\,\nu}(x)\,D^{\nu\,\lambda}(x)\,\partial^{\mu}\,A_{\lambda}(x) + 8\,D^{\dagger}_{\mu\,\nu}(x)\,D^{\nu\,\lambda}(x)\,(\partial^{\mu}\,A_{\lambda}(x) - \partial_{\lambda}\,A^{\mu}(x))\Big]. \quad (4.37)$$

For the derivation of the effective Lagrangian (4.37) we have used the equation of motion

$$\partial_{\lambda}\,D_{\mu\nu}(x)\,+\,\partial_{\mu}\,D_{\nu\lambda}(x)\,+\,\partial_{\nu}\,D_{\lambda\mu}(x)\,=\,0. \quad (4.38)$$

The analogous equation of motion is valid for the conjugated field. The term proportional to k^{λ}_2 contributing to the effective Lagrangian in the form of a divergence of the vector potential of the electromagnetic field $\partial^{\lambda}\,A_{\lambda}(x)$ can be omitted by singling out the Lorentz gauge constraint for the electromagnetic potential, i.e. $\partial^{\lambda}\,A_{\lambda}(x)\,=\,0$.

Collecting like terms in (4.37) we get

$$\mathcal{L}_{\rm Fig.\ 2d}(x)\,=\,(-\,i\,e)\,\frac{g^2_{\rm T}}{4\,\pi^2}\,\frac{1}{M^2_{\rm D}} \times \Big[\frac{8}{3}\,(b\,+\,a-1)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,\partial^{\mu}\,A_{\lambda}(x) + \frac{8}{3}\,(b-a-3)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,\partial_{\lambda}\,A^{\mu}(x) + \frac{8}{3}\,a\,\partial_{\lambda}\,D^{\dagger\lambda\,\nu}(x)\,D_{\nu\mu}(x)\,A^{\mu}(x) + \frac{8}{3}\,a\,D^{\dagger\mu\nu}(x)\,\partial_{\lambda}\,D^{\lambda\nu}(x)\,A_{\mu}(x)\Big]. \quad (4.39)$$

The third and the fourth terms can be reduced by applying the equation of motion

$$\partial_{\lambda}\,D^{\lambda\,\nu}(x)\,=-M^2_{\rm D}\,D^{\nu}(x)$$

and analogous for the conjugated field. Then, setting

$$b\,+\,a-1\,=-b\,+\,a\,+\,3 \quad (4.40)$$

we obtain $b\,=\,2$ and bring up the effective Lagrangian (4.39) to the following irreducible form

$$\mathcal{L}_{\rm Fig.\ 2d}(x) = i\,e\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,a\,[-\,D^{\dagger}_{\mu\nu}(x)\,A^{\nu}(x)\,D^{\mu}(x) + D^{\mu\nu}(x)\,A_{\nu}(x)\,D^{\dagger}_{\mu}(x)] + i\,e\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,(1\,+\,a) \times D^{\dagger}_{\mu\nu}(x)\,D^{\nu\,\lambda}(x)\,(\partial_{\lambda}\,A^{\mu}(x) - \partial^{\mu}\,A_{\lambda}(x)). \quad (4.41)$$

The first two terms define the finite contributions to the renormalization constant of the wave function of the deuteron, whereas the last term coincides with the well–known phenomenological interaction $\mathcal{L}_{\rm A}(x)$ introduced by Aronson [19]

$$\mathcal{L}_{\rm A}(x) = i\,e\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,(1+a)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_{\lambda}{}^{\mu}(x) \quad (4.42)$$

for the description of the charged vector field coupled to an electromagnetic field.

Thus, we have shown that the anomaly of the one–nucleon loop triangle VTT–diagram, where V and T stand for the vector and tensor vertices determined by the Dirac matrices γ^{α} and $\sigma^{\mu\nu}$, respectively, calculated at leading order in the large N_C expansion defines fully the phenomenological Aronson Lagrangian describing the deuteron coupled to an external electromagnetic field.

4.3 The magnetic dipole and electric quadrupole moments of the deuteron

The effective Lagrangian describing both the magnetic dipole and electric quadrupole moments of the deuteron is determined by the sum of $\mathcal{L}_{\rm Fig.\ 2c}(x)$ and $\mathcal{L}_{\rm Fig.\ 2d}(x)$ given by (4.28) and (4.41), respectively, and reads

$$\delta\mathcal{L}^{\rm el}_{\rm eff}(x) = i\,e\,\frac{bg^2_{\rm V}-4ag^2_{\rm T}}{6\,\pi^2}\,D^{\dagger}_{\mu\nu}(x)\,A^{\nu}(x)\,D^{\mu}(x) - i\,e\,\frac{bg^2_{\rm V}-4ag^2_{\rm T}}{6\,\pi^2}\,D^{\mu\nu}(x)\,A_{\nu}(x)\,D^{\dagger}_{\mu}(x) + i\,e\,(3-2b)\,\frac{g^2_{\rm V}}{6\,\pi^2}\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,F^{\mu\nu}(x) + i\,e\,(1\,+\,a)\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_{\lambda}{}^{\mu}(x), \quad (4.43)$$

where a and b are arbitrary parameters related to ambiguities of the one–nucleon loop diagrams with respect to a shift of a virtual nucleon momentum. We consider them as free parameters of the approach [15].

In order to fix these parameters it is convenient to write down the total effective Lagrangian of the physical deuteron coupled to an external electromagnetic field

$$\mathcal{L}^{\rm el}_{\rm eff}(x) = -\frac{1}{2}\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + M^2_{\rm D}\,D^{\dagger}_{\mu}(x)D^{\mu}(x) + i\,e\,\frac{bg^2_{\rm V}-4ag^2_{\rm T}}{6\,\pi^2}\,D^{\dagger}_{\mu\nu}(x)\,A^{\nu}(x)\,D^{\mu}(x) - i\,e\,\frac{bg^2_{\rm V}-4ag^2_{\rm T}}{6\,\pi^2}\,D^{\mu\nu}(x)\,A_{\nu}(x)\,D^{\dagger}_{\mu}(x) + i\,e\,(3-2b)\,\frac{g^2_{\rm V}}{6\pi^2}\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,F^{\mu\nu}(x) + i\,e\,(1\,+\,a)\,\frac{2g^2_{\rm T}}{3\pi^2}\,\frac{1}{M^2_{\rm D}}\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_{\lambda}{}^{\mu}(x). \quad (4.44)$$

Two terms having the structure $D^{\mu\nu}(x)A_\mu(x)D^\dagger_\nu(x)$ and $D^\dagger_{\mu\nu}(x)A^\mu(x)D^\nu(x)$ should describe the interaction of the deuteron with an external electromagnetic field included by a minimal way, whilst the last two terms are responsible for the non–trivial contributions to the magnetic dipole and electric quadrupole moments of the deuteron. In terms of the parameters of the effective interactions (4.44) the magnetic dipole moment $\mu_{\rm D}$, measured in nuclear magnetons, and the electric quadrupole moment $Q_{\rm D}$, measure in fm^2, of the deuteron are given by

$$\mu_{\rm D} = (1 + a)\frac{g^2_{\rm T}}{3\pi^2} + (3 - 2\,b)\,\frac{g^2_{\rm V}}{12\pi^2},$$
$$Q_{\rm D} = \left[(2 + 2\,a)\,\frac{g^2_{\rm T}}{3\pi^2} - (3 - 2\,b)\,\frac{g^2_{\rm V}}{6\pi^2}\right]\frac{1}{M^2_{\rm D}} \quad (4.45)$$

at the constraint

$$b\,\frac{g^2_{\rm V}}{6\pi^2} - 2\,a\,\frac{g^2_{\rm T}}{3\pi^2} = 1 \quad (4.46)$$

reducing the first two terms in effective Lagrangian (4.43) to the standard minimal form which can be obtained from the effective Lagrangian of the free deuteron field by the shift $\partial_\mu D_\nu(x) \to (\partial_\mu + i\,e\,A_\mu(x))D_\nu(x)$.

Retaining the former relation between the electric quadrupole moment and the coupling constant $g_{\rm V}$, $Q_{\rm D} = 2g^2_{\rm V}/\pi^2 M^2_{\rm D}$ [15] that gives $g_{\rm V} = 11.319$, we express the parameters a and b in terms of the coupling constants $g_{\rm V}$, $g_{\rm T}$ and the magnetic dipole moment $\mu_{\rm D}$:

$$a = -1 + \frac{3}{2}\,\frac{\pi^2}{g^2_{\rm T}}\left(\mu_{\rm D} + \frac{g^2_{\rm V}}{\pi^2}\right), \quad b = \frac{9}{2} - \frac{3\pi^2}{g^2_{\rm V}}\,\mu_{\rm D}. \quad (4.47)$$

Substituting (4.47) in (4.46) we get the relation between coupling constants $g_{\rm V}$, $g_{\rm T}$ and the magnetic dipole moment $\mu_{\rm D}$

$$g_{\rm T} = \sqrt{\frac{3}{8}\,g^2_{\rm V} + \frac{3}{2}\,\pi^2\left(1 + \frac{3}{2}\,\mu_{\rm D}\right)} = 0.799\,g_{\rm V}. \quad (4.48)$$

The numerical value $g_{\rm T} = 0.799\,g_{\rm V}$ we obtain at $g_{\rm V} = 11.319$ [15], $\mu_{\rm D} = 0.857$ [22] and $N_C = 3$. The sign of the coupling constant $g_{\rm T}$ should coincide with the sign of the coupling constant $g_{\rm V}$. For the opposite sign the binding energy $\varepsilon_{\rm D}$ given by (4.8) becomes negative that means the absence of the bound neutron–proton state with quantum numbers of the deuteron.

For the evaluation of the binding energy $\varepsilon_{\rm D}$ determined by (4.8) we should keep only the leading contribution to the coupling constant $g_{\rm T}$ in the large N_C expansion, i.e.

$$g_{\rm T} = \sqrt{\frac{3}{8}}\,g_{\rm V} + O(1/\sqrt{N_C}). \quad (4.49)$$

Substituting this relation into (4.8) we can describe the experimental value of the binding energy of the deuteron $\varepsilon_{\rm D} = 2.225\,{\rm MeV}$ at the cut–off $\Lambda_{\rm D} = 46.172\,{\rm MeV}$. The spatial region of virtual nucleon field fluctuations forming the physical deuteron related to this value of the cut–off $1/\Lambda_{\rm D} \sim r_{\rm D} = 4.274\,{\rm fm}$ agrees good with the experimental value of the radius of the deuteron $r_{\rm D} = (4.31895 \pm 0.00009)\,{\rm fm}$ [22]. This result confirms estimates obtained in [15].

The effective Lagrangian of the deuteron field coupled to an external electromagnetic field is given by

$$\begin{aligned}
&{\cal L}^{\rm el}_{\rm eff}(x) = \\
&-\frac{1}{2}\,[(\partial_\mu - i\,e\,A_\mu(x))D^\dagger_\nu(x) - (\partial_\nu - i\,e\,A_\nu(x))D^\dagger_\mu(x)]\\
&\times[(\partial^\mu + i\,e\,A^\mu(x))D^\nu(x) - (\partial^\nu + i\,e\,A^\nu(x))D^\mu(x)]\\
&+M^2_{\rm D}\,D^\dagger_\mu(x)D^\mu(x)\\
&+i\,e\left(\mu_{\rm D} - \frac{1}{2}\,Q_{\rm D}\,M^2_{\rm D}\right)D^\dagger_\mu(x)\,D_\nu(x)\,F^{\mu\nu}(x)\\
&+i\,e\left(\mu_{\rm D} + \frac{1}{2}\,Q_{\rm D}\,M^2_{\rm D}\right)\frac{1}{M^2_{\rm D}}\,D^\dagger_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_\lambda{}^\mu(x).
\end{aligned} \quad (4.50)$$

The term of order $O(e^2)$ can be also derived in the NNJL model by using shift ambiguities of one–nucleon loop diagrams. This term is required by the electromagnetic gauge invariance of the effective Lagrangian of the deuteron field coupled to an external electromagnetic field, but it does not affect on the electromagnetic parameters of the deuteron which are of order $O(e)$.

5 Conclusion

We have shown that the Nambu–Jona–Lasinio model of light nuclei or the NNJL model as well as the ENJL model with chiral $U(3)\times U(3)$ symmetry [4–11] is motivated by QCD. The NNJL model describes low–energy nuclear forces in the nuclear phase of QCD in terms of one–nucleon loop exchanges. One–nucleon loop exchanges provide a minimal way of the transfer of nucleon flavours from an initial to a final nuclear state and allow to take into account contributions of nucleon–loop anomalies. These anomalies are related to high–energy fluctuations of virtual nucleon fields, i.e. the $N\bar{N}$ fluctuations, and fully determined by one–nucleon loop diagrams [29–31]. The dominance of contributions of one–nucleon loop anomalies to effective Lagrangians describing low–energy interactions of the deuteron coupled to itself, nucleons and other particles we justify within the large N_C expansion in QCD with $SU(N_C)$ gauge group at $N_C \to \infty$. It is well–known that anomalies of quark–loop diagrams play an important role for the correct description of strong low–energy interactions of low–lying hadrons [4–11]. We argue an important role of nucleon–loop anomalies for the correct description of low–energy nuclear forces in the nuclear physics.

It should be emphasized that nucleon–loop anomalies can be interpreted as non–trivial contributions of the non–perturbative quantum vacuum – the nucleon Dirac sea [32]. In nuclear physics the influence of the nucleon Dirac sea on low–energy properties of finite nuclei has been analysed within quantum field theoretic approaches in the one–nucleon loop approximation [33]. Unfortunately, in these approaches contributions of one–nucleon loop anomalies have not been taken into account. The NNJL model allows to fill this blank.

For the derivation of the NNJL model from the first principles of QCD we distinguish three non–perturbative phases of QCD: 1) the low–energy quark–gluon phase (low–energy QCD), 2) the hadronic phase and 3) the nuclear phase. Skipping over the intermediate low–energy quark–gluon phase by means of the integration over high– and low–energy quark and gluon fluctuations one arrives at the hadronic phase of QCD containing only local hadron fields with quantum numbers of mesons and baryons coupled at energies below the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$. The couplings of low–lying mesons with masses less than the SBχS scale to low–lying octet and decuplet of baryons can be described by Effective Chiral Lagrangians with chiral $U(3) \times U(3)$ symmetry.

Integrating in the hadronic phase of QCD over heavy hadron degrees of freedom with masses exceeding the SBχS scale one arrives at the nuclear phase of QCD which characterizes itself by the appearance of bound nucleon states – nuclei. At low energies the result of integration over heavy hadron degrees of freedom can be represented in the form of phenomenological local many–nucleon interactions. Some of these interactions are responsible for creation of many–nucleon collective excitations which acquire the properties of observed nuclei through nucleon–loop and low–lying meson exchanges. This effective field theory describes nuclei and processes of their low–energy interactions by considering nuclei as elementary particles represented by local interpolating fields.

Following this scenario of the description of nuclei and their low–energy interactions from the first principles of QCD the deuteron should be produced in the nuclear phase of QCD by a phenomenological local four–nucleon interaction as the Cooper np–pair with quantum numbers of the deuteron. The low–energy parameters of the physical deuteron, i.e. the binding energy, the magnetic dipole $\mu_{\rm D}$ and electric quadrupole $Q_{\rm D}$ moments and so, the Cooper np–pair acquires through one–nucleon loop exchanges. We have shown that the main part of the kinetic term of the effective Lagrangian of the free physical deuteron field is induced by the contribution of high–energy (short–distance) fluctuations of virtual nucleon fields related to the anomaly of the one–nucleon loop VV–diagram with two vector vertices.

In turn, the magnetic dipole $\mu_{\rm D}$ and electric quadrupole $Q_{\rm D}$ moments of the physical deuteron are fully determined by high–energy (short–distance) fluctuations of virtual nucleon fields related to the anomalies of the triangle one–nucleon loop VVV– and VTT–diagrams. Thus, high–energy (short–distance) fluctuations of virtual nucleon fields related to anomalies of one–nucleon loop diagrams play a dominant role for the correct description of electromagnetic properties of the physical deuteron in the NNJL model.

As regards low–energy (long–distance) fluctuations of virtual nucleon fields they give a significant contribution only to the binding energy of the deuteron $\varepsilon_{\rm D}$. The strength of low–energy (long–distance) fluctuations of virtual nucleon fields is restricted by the cut–off $\Lambda_{\rm D} = 46.172\,\mathrm{MeV}$. The spatial region of virtual nucleon field fluctuations forming the physical deuteron related to this value of the cut–off $1/\Lambda_{\rm D} \sim r_{\rm D} = 4.274\,\mathrm{fm}$ agrees good with the experimental value of the radius of the deuteron $r_{\rm D} = (4.31895 \pm 0.00009)\,\mathrm{fm}$ [22]. This confirms our estimates obtained in [15].

It is well–known that in the potential model approach to the description of the deuteron the electric quadrupole moment of the deuteron $Q_{\rm D}$ is caused by nuclear tensor forces which are of great deal of importance for the existence of the deuteron as a bound np–state [34].

The proportionality of the coupling constant of the phenomenological local four–nucleon interaction (3.3), responsible for creation of the Cooper np–pair with quantum numbers of the deuteron, and the binding energy of the deuteron $\varepsilon_{\rm D}$ (3.20) to the electric quadrupole moment $Q_{\rm D}$ testifies an important role of nuclear tensor forces for the formation of the deuteron in the NNJL model.

To the evaluation of one–nucleon loop diagrams defining effective Lagrangians describing processes of low–energy interactions of the deuteron coupled to itself and an electromagnetic field we apply expansions in powers of the momenta of interacting particles and keep only leading terms of the expansions. This approximation can be justified in the large N_C expansion. Indeed, in QCD with the $SU(N_C)$ gauge group at $N_C \to \infty$ the nucleon mass is proportional to the number of quark colours [17]: $M_{\rm N} \sim N_C$. Since for the derivation of effective Lagrangians describing the deuteron and amplitudes of low–energy nuclear processes all external momenta of interacting particles should be kept off–mass shell, the masses of virtual nucleon fields are larger compared with the external momenta. An expansion of one–nucleon loop diagrams in powers of $1/M_{\rm N}$ giving an external momentum expansion corresponds to the expansion in powers of $1/N_C$. In this case the leading order in the large N_C expansion gives the leading order contributions in the expansion in powers of external momenta of interacting particles. We should emphasize that anomalous contributions of one–nucleon loop diagrams are determined by the least powers in external momentum expansions. Thereby, the dominance of contributions of nucleon–loop anomalies to effective Lagrangians describing low–energy nuclear forces in the NNJL model is fully supported by the large N_C expansion. The accuracy of this approximation is rather high. Indeed, the real parameter of the expansion of one–nucleon loop diagrams is $1/M^2_{\rm N} \sim 1/N^2_C$ but not $1/M_{\rm N} \sim 1/N_C$. Thereby, next–to–leading corrections should be of order $O(1/N^2_C)$.

The inclusion of the interaction of the deuteron field with the tensor nucleon current (4.1) has given a possibility of the self-consistent description of the electromagnetic properties of the deuteron, the magnetic dipole moment $\mu_{\rm D}$ and the electric quadrupole moment $Q_{\rm D}$, in terms of effective interactions of the Corben–Schwinger and Aronson types induced by one–nucleon loop diagrams. By fitting the experimental values of the magnetic dipole moment $\mu_{\rm D} = 0.857$, measured in nucleon magnetons $\mu_{\rm N} = e/2M_{\rm N}$, and the electric quadrupole moment $Q_{\rm D} = 0.286$, measured in $\rm fm^2$, supplemented by the requirement of the electromagnetic gauge invariance of the effective Lagrangian of the deuteron field coupled to an external electromagnetic field we have got the relation between the coupling constants $g_{\rm V}$ and $g_{\rm T}$: $g_{\rm T} = 0.799\,g_{\rm V}$ calculated at $N_C = 3$. At leading order in the large N_C expansion we get $g_{\rm T} = \sqrt{3/8}\,g_{\rm V} + O(1/\sqrt{N_C})$. This relation agrees good with that obtained in [14] (see (16) of [14]). Due to this relation the experimental value of the binding energy of the deuteron can be described by the cut–off $\Lambda_{\rm D} = 46.172\,{\rm MeV}$. This corresponds to the spatial region of virtual nucleon field fluctuations forming the physical deuteron $1/\Lambda_{\rm D} \sim r_{\rm D} = 4.274\,{\rm fm}$ agreeing good with the experimental value of the radius of the deuteron $r_{\rm D} = (4.31895 \pm 0.00009)\,{\rm fm}$ [22].

For further applications of the NNJL model to the description of low–energy nuclear reactions of astrophysical interest we anticipate the results in agreement with those obtained in [35,36].

The quantum field theoretic scenario to treating nuclei as many–nucleon collective excitations induced by phenomenological local many–nucleon interactions allows a plain extension of the NNJL model by the inclusion of light nuclei ^{3}He, ^{3}H and ^{4}He as three– and four–nucleon collective excitations. The binding energies and other low–energy parameters of these excitations should be determined through nucleon–loop and low–lying meson exchanges.

On this way it is important to notice that the spinorial structure of the operators of three–nucleon densities coupled to the ^{3}He and the ^{3}H is very much restricted. One can show that only the three–nucleon densities $[\bar{p^c}(x)\gamma^\mu\gamma^5 p(x)]\gamma_\mu n(x)$ and $[\bar{n^c}(x)\gamma^\mu\gamma^5 n(x)]\gamma_\mu p(x)$ can lead to the appearance of the bound ^{3}He and ^{3}H state, respectively. At the quantum field theoretic level this result explains a well–known experimental fact of the compensation of spins and magnetic dipole moments of pp and nn pairs inside nuclei which has been put into the foundation of the shell–model of nuclei [34].

The extension of the NNJL model by the inclusion of ^{3}He, ^{3}H and ^{4}He would give a possibility to analyse within the NNJL model the reactions of the p–p chain [37] started with the reaction $\rm p + p \to D + e^+ + \nu_e$ and to apply the extended version of the NNJL model to the description of the reactions $\rm p + D \to {}^3He + \gamma$, $\rm p + {}^3He \to {}^4He + e^+ + \nu_e$ and so on.

Chiral perturbation theory can be naturally incorporated into the NNJL model [35] in terms of Effective Chiral Lagrangians with chiral $U(3)\times U(3)$ symmetry describing low–lying baryons and mesons interacting at low energies [4–11].

The quantum field theoretic description of the deuteron within the NNJL model can be also of use for the analysis of the properties of dibaryons. Indeed, following Oakes [38] the deuteron can be considered as a component of the $SU(3)_{\rm flavour}$ decuplet $\tilde{\bf 10}_f$ of dibaryons with $Y = 2$ and $I = 0$, where Y and I are the hypercharge and the isotopical spin, respectively. In the chiral limit the binding energies, the magnetic dipole and electric quadrupole moments of dibaryons of the decuplet $\tilde{\bf 10}_f$ should be equal. The splitting of the parameters of the components of the decuplet $\tilde{\bf 10}_f$ can be obtained within Chiral perturbation theory incorporated into the NNJL model.

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